GRADED ARITHMETICS

BOOK THREE—GRADE IV

CHANCELLOR

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Received May 15, 1903.

Room 16, Liberty School. Geo. Windom.

1. Multiply 34601 by 385: and prove the answer.

```
    34601          385)13321385(34601
      385              1155
   173005               1771        Answer
  276808                1540        13321385
 103803                  2313
 13321385                2310
                          385
                          385
```

2 What is the ratio of two years to four months?

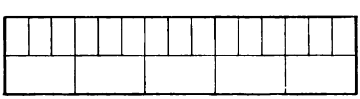
1 yr. = 12 mo.

2 yr. = 12 mo. x 2 = 24 mo.

4 mo. : 24 mo. = 1 : x   4x = 24   x = 6

Answer: The ratio is six.

3 Show by a drawing that $\frac{1}{5} = \frac{3}{15}$



15ths. $\frac{3}{15} = \frac{1}{5}$

5ths.

CHILDREN'S ARITHMETICS BY GRADES

GLOBE SERIES

THIRD BOOK

FOURTH YEAR

INTERMEDIATE PRINCIPLES

BY

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SUPERINTENDENT OF SCHOOLS, BLOOMFIELD, N.J.

GLOBE SCHOOL BOOK COMPANY
NEW YORK AND CHICAGO

M. P. 1

"The merely oral method does not suffice for the production of the independent scholar. The text-book method holds the pupil responsible for mastering critically the printed page. It requires him to make alive again its thoughts and perceptions. This training in the critical study of the printed page fits him for the work of independent investigation."

W. T. HARRIS, LL.D.
United States Commissioner of Education.

Address on Horace Mann, N. E. A., Buffalo, 1896.

MANHATTAN PRESS
474 WEST BROADWAY
NEW YORK

PREFACE

This book is for boys and girls who can use numbers into the thousands in addition and subtraction, who know what ratio is and understand something about fractions, who know some facts of measures, and who have studied the multiplication tables. For these boys and girls there is here about a year's work, chiefly in practice, partly in extension of knowledge.

The new topics introduced here are direct outgrowths of earlier topics, and are presented in their simplest forms and relations. Ratio develops into proportion, and proportion into equation. The common fraction is correlated with percentage, with the decimal, and with ratio and proportion. Factoring is centered upon dividing, and cancellation is centered upon both. The method of long division is explained. A few pages are devoted to each of two topics, greatest common divisor and least common multiple. But the emphasis of the book is upon the fundamental operations of addition and subtraction, multiplication and division for drill, and upon fractions and measures for development of arithmetical knowledge and skill.

This book is more nearly topical in arrangement than either of the two earlier books of this Series; but an essentially spiral character is secured by frequent reviews. Subjects are not exhausted before we pass to new subjects or return to old subjects displayed in new matter.

It is sound child-psychology to believe that boys and girls in fourth, fifth, and sixth grades have singular power and even pleasure in undergoing the exercises of drill in any and every subject. It is fortunate that there is such a period in the

mental and physical life as that passed by children usually in their fourth, fifth, and sixth grades at school. It is unfortunate that sufficient advantage of this age in life has not yet been taken by the text-book writers. In particular, in arithmetic we have many excellent books and monographs discussing methods, devices, and exercises in teaching the elementary facts of number from one to twenty, and very many books discussing and exemplifying arithmetic as a science. Most of these latter books are logical, but few are psychological. They are treatises upon arithmetic or manuals for adult teachers rather than text-books for young students. In a bibliography of many hundred titles one finds not ten books and essays which even try to present, or to demonstrate proper methods of presenting, to boys and girls nine to twelve years of age such fundamental and intermediate operations as are indicated in this book. Our pedagogical philosophers have clear and correct ideas of the minds of six year old children; and our text-book writers have at least scientifically arranged the topics of arithmetic for fifteen year old boys and girls. But I fail to find any considerable number of authors who have deliberately set before themselves the problems of the minds of the boys and girls who are likely to need such a book as this.

On the other hand, we find year by year our business men saying that the boys and girls who leave school at the ages of twelve to fourteen know nothing accurately in arithmetic, not even the fundamental operations. This is a curious fact in view of the other fact that the very power of the ten to twelve year old child is to undergo drill. This book is an effort to present interesting forms of such drill as will develop quickness and accuracy.

It is not meant that we are to question the principle that "the problem is the unit of arithmetic." It is meant that success in the problem is conditioned by mere mechanical facility and correctness in number-manipulation. There ought to be many problems in considerable variety in every arith-

metic text-book. Every characteristic problem ought to be thoroughly studied and completely understood, and this as early in the child's life as possible.

I sympathize strongly, as this series shows, with the ratio-apostles who teach ratio "early and often." I see in ratio the true key to the fraction which is otherwise the child's arithmetical "bugaboo." But I am far from believing that we can profitably thrust many of the complications of percentage and interest, of denominate numbers, even of fractions, into grades lower than those in which they are nowadays usually found in our American common schools. Hence in this third book of the series designed for children's manuals I have seized as a true educational opportunity the time of teachers and students for many exercises in the simple, ever useful principles of addition, subtraction, multiplication, and division, and in the elements of common fractions, ratio, proportion, percentage, and decimals. It has often been said that "science is exact measurement." These principles are the instrumentalities of exact measurement, whether in the bookkeeping of a commercial business, or in the speculations of mathematical astronomy, or in the researches of quantitative chemistry.

In work in such a grade as this book is designed to serve it is unquestionable that the boys and girls need a text-book to study. It is very easy for teachers to over-develop the arithmetical instruction of boys and girls, one of whose greatest joys is, if they have been rightly taught, in finding out things for themselves and in setting their own tasks. We are here at a stage in advance of that in which it is constantly necessary to renew in the pupil's mind the sense of the facts that figures are merely the symbols of numbers and that numbers always relate to real things. We can now deal with figures, manipulating them usually without consciousness of what they represent, in exactly the spirit of the bank employee who forgets that the coins and bank notes he handles are money with purchasing power, and treats his cash, as he ought, in isolation from its concrete values.

It may be argued that this is the disciplinary or cultural view of arithmetic rather than the utilitarian. If so, I hold it because of my understanding of the actual powers and interests of children. The supremacy of the human creature is due to his long infancy, to his long preparation before the responsibilities of self-dependence roll in upon him. In the light of this fact, it is both necessary and desirable to postpone, to the years when the child can understand them, those utilities of education which involve the minute details of practical life. The greatest of all the world's utilities is an affirmative, honest, accurate human mind. Arithmetic, even though chiefly disciplinary, is perfectly justified in its forms and methods if it tends to produce an active, clear, and careful mind.

Author and publishers desire to acknowledge the very helpful suggestions of Superintendent Frank E. Spaulding, Ph.D., of Passaic, N.J., in reviewing proofs of these pages.

W. E. C.

Bloomfield, N.J.,
August 3, 1901.

SUGGESTIONS TO TEACHERS

1. The preface explains the general purpose of the book.

2. Read the book itself. The purposes of certain special features appear only when considered in relation to other features.

3. Read also Book II of this Series, which shows what pupils of this grade may be expected to know, or at least to have studied. Whenever the pupils of a class seem weak in matters necessarily preliminary to the grade in which they are, it is essential to review these matters both in instruction and in the pupils' own work. But never expect a class as a whole to know accurately and thoroughly every topic in any subject. Aim for perfect knowledge and proficiency, and one will secure surprisingly good results from some pupils. It is easy to waste time in undue drill of the whole class or in undue attention to individuals. It is very easy in arithmetic to spend an unnecessary amount upon single topics. A book upon the spiral plan, but which does not neglect definite study of each topic as it is introduced or renewed, affords at frequent intervals an opportunity to find out the facts as to the conditions of the class and of its individual pupils.

4. It is unwise to ask children always to solve examples by the method of the book or by that of the teacher. Often children see through problems in ways distinctly individual. If their solutions are logical, correct, clear, and brief, we ought not only to accept, but also to welcome them.

5. One kind of exercises is more valuable than any other in arithmetic: setting by pupils of their own problems and finding the solutions. Exactly as we encourage in our language courses writing by the pupils themselves of various

kinds of compositions, so ought we to encourage the making of their own examples. Such pages as are given here for drill in addition, subtraction, multiplication, and division are merely illustrations of exercises which may be set by the pupils for themselves. Every week there should be a period in the program for invention of problems by the pupils. One caution is to be observed. It is best for the teacher to inspect the problems to see that they are capable of solution. This invention of problems makes the pupils independent of printed answers and stimulates them to try proofs for themselves.

6. By the time the class is near the end of this book the multiplication tables should be known exactly, and such facts of the tables of weights and measures as have been introduced should be known systematically in tabular order. It is undesirable to pay much attention to the words of arithmetical rules and definitions at this stage lest the forms conceal the principles. Reasoning is the soul of arithmetic. Memory itself is simply an established process of thinking.

7. Incidentally facts of the business world, as the children see it for themselves, may well be used. What facts will depend upon the neighborhood of the school and of the children's homes. These facts may be used as the bases of their invention of problems. This book uses merely standard facts such as may be seen anywhere. Continued observation of local facts will result in the accumulation of great stores of knowledge for use from time to time in instruction and drill. Among the facts that ought to be known by children and are valuable as material for making problems are such as these; viz.:

(*a*) Railroad fares are usually three cents per mile. Children pay half fare. What are the costs for familiar distances? for "excursion" and "return" tickets? to great centers of population?

(*b*) Street-car fares are usually five cents. Children pay three cents. How many miles may one ride for one fare? Are

several tickets sold at lower rates? Are special school children's tickets sold?

(*c*) Make market lists of prices for provisions, meats, groceries, shoes, dry goods, clothes, carpets, furniture, etc. Place these lists on the blackboard, and use them both in written and in oral work.

(*d*) Take imaginary trips, singly or in parties, paying all expenses, — fares, food, purchases, admissions to places of amusement or recreation, etc.

(*e*) As far as possible, get the material from the children, but have it accurate.

8. It is unquestionable that a great variety of problems and exercises tends to make the principles themselves very clear. But avoid too much consecutive repetition of processes. We can secure the memory of processes best by variety of associations. Hence, though we ought to make each topic clear at its first presentation, we ought not to dwell too long upon it, but to go forward to the next, returning occasionally to refresh and to enrich our former knowledge.

9. Neatness in writing tends to accuracy in all exercises. Let us encourage excellent work with pen, pencil, or chalk crayon by commending it. Poor work may be due to defective eyesight, which causes increasing trouble with every added year of life, or the eyes may not be in focus with each other. Blackboard work ought to be clear and large. The figures written by most school children are too small and cramped.

10. In working through such a book as this it is well for a teacher to remember that in the early part of a term or year some problems may prove too hard for the pupils that they will be able to solve easily later in the term or in the year. If the problems are too easy, it is a simple matter to increase the difficulties by adding another step or by dealing with larger figures. While the class is studying the topics, the teacher is studying the class; and while the class improves in industry and ability and technical knowledge, the teacher improves in instruction and in assignment of work through

greater familiarity with the needs and powers of the individuals in the class.

11. In the practical use of this book a greater or a less proportion of the problems will be found too difficult for even the best pupils upon the first consideration of them.

(*a*) Sometimes it will be found advisable to give oral instruction regarding a problem.

(*b*) Many of the class will be able to return a few weeks later to a hitherto unsolved problem with power to solve it unaided by the teacher.

(*c*) Where pupils fail upon many problems, the failure may be due to inability properly to reason out in the imagination both the processes and their concrete bases in ratio and counting. Such pupils need not always more teaching in elementary number and measurement, but more often exercising of their own mental activity by methods suited to their present age in this grade.

A book in which every problem is easy for nearly every child is too easy to give proper inducement for earnest, steady, upbuilding effort on the part of its students.

12. The plan here has been to make a systematic outline, leaving to the teacher's judgment of the daily needs of the class exactly the number of problems and upon exactly what pages they should be for the ever insistent "next lesson." Something new for curiosity and a good deal that is old for power and for skill make the right combination for lessons in any subject. From a half page to a page will be usually a reasonable lesson, including both class and home work.

CONTENTS

INTRODUCTORY REVIEW

1. What is the ratio of one day to a week? of one month to a day? of three hours to a day?

2. John worked 4 days for \$2, and Mary, his older sister, 6 days for \$9. What was the ratio of John's day's wages to Mary's?

3. A milkman sold $\frac{2}{3}$ of a 3-gallon can of milk for 40¢. What was the price per quart?

4. Alice spent a half dollar for a doll, a quarter dollar for an iron savings bank, and a dime for a half yard of blue silk ribbon. How much did she spend in all?

5. The schoolhouse in the village of Williamsport was built in the year MDCCCLXXVI. What year was that as expressed in words?

6. What is the area of a house lot 60 feet by 150 feet?

7. A farmer bought 30 cows for \$1200. What was the average price of each? At this rate, how much would 60 cows have cost? 75? 100?

8. A boy paid 10¢ each for 7 marbles and 5¢ each for 5 other marbles, then 20¢ for 40 more. How many marbles had he in all? What was the cost of all? What was the ratio of the price of each of the first 7 to that of each of the last 40?

REVIEW

1. Subtract:

20,189	$1.59	103,862
14,362	.68	94,342

2. Add:

a.	b.	c.	d.
62	839	$30.81	402,361
39	227	1.05	329,075
47	835	.10	436,829
83	426	32.	603,021
29	121	2.64	329,034

3. What is 2% of $500? of $1000?

4. Find the circumference of a circle 3 yd. in radius.

5. What is the ratio of $\frac{3}{8}$ to $\frac{3}{16}$? to $\frac{3}{4}$?

6. In 100 gills are how many pints?

7. How many gallons are in 4 bbl. of oil?

8. A plot of ground was 60 ft. × 200 ft. It was bought for $\$\frac{1}{2}$ per sq. ft. What was its cost?

9. How many cords of wood are there in a pile of wood, 4 ft. × 8 ft. × 16 ft.?

10. Write the Table of Long Measure.

11. Read 10.89; 502.36; 1.31; 1000.05.

12. Draw one music measure indicating a half note, a quarter note, and two eighth notes. Are these notes added together equal to one whole note?

13. Multiply 362 by: 51; 28; 37; 64.

14. Divide 1287 by: 21; 33; 48; 95.

15. A boy walked 10 miles one day, 18 miles the next, 7 the next, and 19 the next. He was then just $\frac{1}{2}$ of the way home. How far was his starting point from his house?

16. Draw a *triangle;* a *square;* a *circle.*

REVIEW

1. Add $3.50, $2.40, a dime, a quarter, a half dollar.

2. John spent ninety cents for three rubber tennis balls. What was the price of each?

3. Which is more money, 3000 cents or 300 dimes?

4. What is the ratio of 2 cwt. to 1 T.?

5. How many degrees of temperature are there between the freezing of water and blood heat? between 100° and the boiling point of water? between zero and freezing?

6. How does a thermometer tell the degrees of heat?

7. How many days has "leap year"?

8. How many ounces are in 4 lb.? in 5 lb.? in 10 lb.?

9. Multiply $3.25 by 4; 5; 6; 8.

10. A boy went to a clothing store with a ten-dollar bill. He bought a suit of clothes for $7, a hat for $1.25, and a pair of shoes for $2. Did he have enough money in the bill to pay for all these things?

11. Which is farther, 2000 yd. or 1 mi.? By how much?

12. One boy threw a baseball 100 ft. Another threw it 125 ft. What is the ratio of 100 to 125?

13. Which is more, $\frac{5}{18}$ or $\frac{1}{3}$? By how much?

14. What is a *dividend?* a *multiplicand?* a *factor?*

15. What is meant by the *address* of a house?

16. Read: MCM; XCII; XXXIX; XLVI.

17. What is a *pentagon?* a *hexagon?* an *octagon?* a *decagon?* a *rectangle?* a *circumference?*

18. Which is more, 6 mo. or 25 wk.?

19. Tell the Roman numerals for 1902, 1905, 1910, 1915.

20. From 2000 take 1850; 1329; 1111; 975.

ADDITIONS

	I	II	III	IV	V	VI	VII	VIII	IX
A	2	4	5	7	8	1	3	6	9
B	1	3	4	6	7	9	2	5	8
C	3	5	6	8	9	2	4	7	1
D	5	7	8	1	2	4	6	9	3
E	8	1	2	4	5	7	9	3	6
F	4	6	7	9	1	3	5	8	2
G	9	2	3	5	6	8	1	4	7
H	7	9	1	3	4	6	8	2	5
I	6	8	9	2	3	5	7	1	4

1. Add the columns; add the rows.

2. Add every two columns: 24, 13, 35, etc.; 45, 34, 56, etc.

3. Add every two rows, as in 2.

4. Add every three columns: 245, 134, 356, etc.

5. Add every three rows, as in **4**.

6. Add I and II together, using the numbers as units: 2, 4, 1, 3, and so on.

7. Call I and II tens and units of dollars and III and IV tens and units of cents, and add the amounts.

8. Call A and B dollars and C and D cents, and add.

9. Add diagonally from I A to IX I, etc.

10. Multiply together every two numbers by rows, etc.

These exercises may be varied in many ways for both addition and subtraction.

QUESTIONS

1. If one family burns one ton of coal a month and another family burns two tons in half a year, what is the ratio of the coal used by the first family to that used by the second?

2. What is the yearly cost of coal for a family that uses on the average $1\frac{1}{2}$ T. per mo. @ $5 per T? What is the cost to another family using $\frac{2}{3}$ as much?

3. If a barrel of flour weighs 196 pounds, what is the weight of a bag of flour containing one eighth of a barrel?

4. If flour is $4.80 per bbl., what is the cost of $\frac{1}{8}$ bbl.?

5. In a school of 600 children there was an average of 30 children in each room. How many rooms were there?

6. A boy's father bought for him a bicycle costing $25. At the same price, what would the father have to pay for four bicycles for four sons?

7. Which is lower in price per sheep: 40 head of sheep for $320, or 12 head for $144?

8. A man who wished to build a house was offered two different lots of land: one was 60 feet wide, price $900, the other was 40 feet wide, price $800. Which was cheaper per front foot? Make drawing on blackboard.

9. A little boy paid a dime for a dozen marbles, though another boy offered him seven marbles for a nickel. Which marbles were cheaper?

10. One room has 15 square yards of carpet, another has 16 square yards. What may be the dimensions of each room in feet? Make drawings on blackboard.

COUNTING BY LARGE NUMBERS TO 200

13

1. 13, 26, 39, 52, 65, 78, 91, 104, 117, 130, 143, 156, 169, 182, 195.

2. How many 13's are there in each of these numbers?

14

3. 14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154, 168, 182, 196.

4. How many 14's are there in each of the numbers?

15

5. 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, 195.

6. How many 15's are there in each of the numbers?

16

7. 16, 32, 48, 64, 80, 96, 112, 128, 144, 160, 176, 192.

8. How many 16's are there in each of the numbers?

17

9. 17, 34, 51, 68, 85, 102, 119, 136, 153, 170, 187.

18

10. 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, 198.

19

11. 19, 38, 57, 76, 95, 114, 133, 152, 171, 190.

20

12. 20, 40, 60, 80, 100, 120, 140, 160, 180, 200.

24

13. 24, 48, 96, 120, 144, 168, 192.

25

14. 25, 50, 75, 100, 125, 150, 175, 200.

NUMERATION

ORDERS.	Hundred billions.	Ten billions.	Billions.	Hundred millions.	Ten millions.	Millions.	Hundred thousands.	Ten thousands.	Thousands.	Hundreds.	Tens.	Units.
	8	6	5	1	5	9	5	7	3	2	0	4
PLACES.	12th,	11th,	10th,	9th,	8th,	7th,	6th,	5th,	4th,	3d,	2d,	1st,
PERIODS.		4th.			3d.			2d.			1st.	

The number here, 865 159 573 204, is read, eight hundred sixty-five billion, one hundred fifty-nine million, five hundred seventy-three thousand, two hundred four.

It is usual, in writing large numbers, either to point off the periods with commas, 428,532,829, or to leave a space between the periods, 428 532 829. This is simply as a convenience for the eye of the reader.

Only a few centuries ago, words even were not separated by spaces, and sentences were not begun with capitals or marked at the ends with periods.

1. Read 9 291 565 432; 5 827 002 472; 902 873 429 826.

2. The stocks and bonds of the United States Steel Corporation exceed $1 400 000 000. Read these figures.

3. The average distance of the earth from the sun is 91 430 000 miles. Read these figures.

4. If the size of the United States is 2500 miles by 1300 miles, how many square miles does it contain? Since each square mile contains 640 acres, how many acres in all are there?

SUBTRACTION

Any number of numbers may be added at one time; but we can subtract, multiply, and divide only two numbers at any one time.

In subtracting, we call the number from which another number is taken the **minuend**, and the number taken away the **subtrahend**. The result we call the **difference**, or *remainder*.

PROOFS

The *difference* and the *subtrahend* together always equal the *minuend*. $18 - 6 = 12$, $12 + 6 = 18$.

The *minuend* less the *difference* always equals the *subtrahend*. $18 - 6 = 12$, $18 - 12 = 6$.

QUESTIONS

WRITE

Prove all answers in two ways.

1. $1857 - 289 = ?$
2. $10940 - 1111$.
3. $40{,}000 - 28{,}931$.
4. $\$349.27 - \$287.36 = ?$
5. $\$800.00 - \$732.80$.
6. $\$432. - \$381.25$.

7. Mr. William Thompson, when he was twenty-one years of age, inherited $1,297,856, which was given into his possession by the trustees of his father's property. He found that $829,642 was invested in land and buildings. How much was invested in other property, such as railroad shares and government bonds?

8. A great city owed $10,675,431, and the property of all its people was considered worth $231,482,649. What was the difference between all the private property and the public debts?

MULTIPLICATION TABLES

TWOS	THREES	FOURS	FIVES
$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$	$1 \times 5 = 5$
$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$	$2 \times 5 = 10$
$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$	$3 \times 5 = 15$
$4 \times 2 = 8$	$4 \times 3 = 12$	$4 \times 4 = 16$	$4 \times 5 = 20$
$5 \times 2 = 10$	$5 \times 3 = 15$	$5 \times 4 = 20$	$5 \times 5 = 25$
$6 \times 2 = 12$	$6 \times 3 = 18$	$6 \times 4 = 24$	$6 \times 5 = 30$
$7 \times 2 = 14$	$7 \times 3 = 21$	$7 \times 4 = 28$	$7 \times 5 = 35$
$8 \times 2 = 16$	$8 \times 3 = 24$	$8 \times 4 = 32$	$8 \times 5 = 40$
$9 \times 2 = 18$	$9 \times 3 = 27$	$9 \times 4 = 36$	$9 \times 5 = 45$
$10 \times 2 = 20$	$10 \times 3 = 30$	$10 \times 4 = 40$	$10 \times 5 = 50$
$11 \times 2 = 22$	$11 \times 3 = 33$	$11 \times 4 = 44$	$11 \times 5 = 55$
$12 \times 2 = 24$	$12 \times 3 = 36$	$12 \times 4 = 48$	$12 \times 5 = 60$
$13 \times 2 = 26$	$13 \times 3 = 39$	$13 \times 4 = 52$	$13 \times 5 = 65$
$14 \times 2 = 28$	$14 \times 3 = 42$	$14 \times 4 = 56$	$14 \times 5 = 70$
$15 \times 2 = 30$	$15 \times 3 = 45$	$15 \times 4 = 60$	$15 \times 5 = 75$
$16 \times 2 = 32$	$16 \times 3 = 48$	$16 \times 4 = 64$	$16 \times 5 = 80$
$17 \times 2 = 34$	$17 \times 3 = 51$	$17 \times 4 = 68$	$17 \times 5 = 85$
$18 \times 2 = 36$	$18 \times 3 = 54$	$18 \times 4 = 72$	$18 \times 5 = 90$
$19 \times 2 = 38$	$19 \times 3 = 57$	$19 \times 4 = 76$	$19 \times 5 = 95$
$20 \times 2 = 40$	$20 \times 3 = 60$	$20 \times 4 = 80$	$20 \times 5 = 100$

It is often convenient to know the tables from 13 to 20.

QUESTIONS

1. Beginning at 5, count by 15 to 200.

2. Give the Multiplication Table of Fours.

3. A milkman sold 30 pts. of cream at the rate of 40¢ a quart in cash. How much money did he receive?

4. In a certain city there are 12 blocks of buildings to a mile. What is the ratio of 4 of these blocks to a mile of them?

5. In which city are the blocks longer, one which has 16 blocks, or one which has 8 blocks, to the mile?

6. A city block is 400 feet long, and each house lot in it is supposed to be 20 feet wide. How many house lots has such a block?

7. In a certain village it is $\frac{1}{4}$ of a mile from the church to the post office, $\frac{1}{3}$ of a mile from the school to the church, and $\frac{1}{5}$ of a mile from the post office to the school. Which is the longest, and which the shortest distance?

8. If this line ——— represents a mile, how many miles is it from A ———— to ———— B?

A scale is a ratio.

9. Upon a scale of one quarter inch to a foot, how many feet would a real foot represent?

10. Upon the same scale, how many feet would 5 inches represent?

When men plan houses and buildings, they make drawings to scale, sometimes $\frac{1}{4}$ in. to 1 ft., sometimes $\frac{1}{8}$ in., or $\frac{1}{16}$ in., or even $\frac{1}{32}$ in. to 1 ft.

11. If a house is to be 40 feet wide, by which scale would the plans be largest? By which smallest? Why?

MULTIPLICATION TABLES

SIXES	SEVENS	EIGHTS	NINES
$1 \times 6 = 6$	$1 \times 7 = 7$	$1 \times 8 = 8$	$1 \times 9 = 9$
$2 \times 6 = 12$	$2 \times 7 = 14$	$2 \times 8 = 16$	$2 \times 9 = 18$
$3 \times 6 = 18$	$3 \times 7 = 21$	$3 \times 8 = 24$	$3 \times 9 = 27$
$4 \times 6 = 24$	$4 \times 7 = 28$	$4 \times 8 = 32$	$4 \times 9 = 36$
$5 \times 6 = 30$	$5 \times 7 = 35$	$5 \times 8 = 40$	$5 \times 9 = 45$
$6 \times 6 = 36$	$6 \times 7 = 42$	$6 \times 8 = 48$	$6 \times 9 = 54$
$7 \times 6 = 42$	$7 \times 7 = 49$	$7 \times 8 = 56$	$7 \times 9 = 63$
$8 \times 6 = 48$	$8 \times 7 = 56$	$8 \times 8 = 64$	$8 \times 9 = 72$
$9 \times 6 = 54$	$9 \times 7 = 63$	$9 \times 8 = 72$	$9 \times 9 = 81$
$10 \times 6 = 60$	$10 \times 7 = 70$	$10 \times 8 = 80$	$10 \times 9 = 90$
$11 \times 6 = 66$	$11 \times 7 = 77$	$11 \times 8 = 88$	$11 \times 9 = 99$
$12 \times 6 = 72$	$12 \times 7 = 84$	$12 \times 8 = 96$	$12 \times 9 = 108$
$13 \times 6 = 78$	$13 \times 7 = 91$	$13 \times 8 = 104$	$13 \times 9 = 117$
$14 \times 6 = 84$	$14 \times 7 = 98$	$14 \times 8 = 112$	$14 \times 9 = 126$
$15 \times 6 = 90$	$15 \times 7 = 105$	$15 \times 8 = 120$	$15 \times 9 = 135$
$16 \times 6 = 96$	$16 \times 7 = 112$	$16 \times 8 = 128$	$16 \times 9 = 144$
$17 \times 6 = 102$	$17 \times 7 = 119$	$17 \times 8 = 136$	$17 \times 9 = 153$
$18 \times 6 = 108$	$18 \times 7 = 126$	$18 \times 8 = 144$	$18 \times 9 = 162$
$19 \times 6 = 114$	$19 \times 7 = 133$	$19 \times 8 = 152$	$19 \times 9 = 171$
$20 \times 6 = 120$	$20 \times 7 = 140$	$20 \times 8 = 160$	$20 \times 9 = 180$

Write these tables from memory.

REVIEW

1. Count by 18 to 200.

2. What is $\frac{1}{6}$ of 24, 36, 72, 30, 96, 48, 60, 84?

3. Read 32,498,721.

4. Tell the figures for three billion nine hundred million.

5. The sun is over 800,000 miles in diameter. How many miles is it in circumference? $3\frac{1}{7} \times 800{,}000 = ?$

6. The earth is 25,000 miles in circumference. About how many miles is it in diameter? $25{,}000 \div 3\frac{1}{7} = ?$

7. If the United States is 2500 miles wide from New York to San Francisco, what is the ratio of its breadth to the circumference of the earth?

8. Since the earth is 8000 miles in diameter, what is the ratio of its diameter to that of the sun?

9. If a good silver watch costs $8, and a good gold watch $40, how many silver watches can be bought for the price of one gold watch?

10. If the United States has about seventy-five million people, and if China has at least three hundred million, what is the ratio of our population to that of China?

11. If one book cost five dollars, and a set of sixteen books, of a different kind, cost four dollars, what is the ratio of the cost of the first book to that of one of the other books?

12. Mr. Williams bought 100 M. bricks for $800, and sold 60 M. for $540. Did he make or lose per M.?

13. A contractor employed a carpenter at $3 per day, and charged $3.50 in the bill for his work. What was the ratio of the contractor's gain to the workman's wages? to the charge in the bill?

MULTIPLICATION TABLES

TENS

To multiply by ten, add a zero at the right of the figures which represent the number. This advances each figure to an order one place higher. Each place shows a multiple by ten of the next lower place.

ELEVENS

To multiply by eleven, in the case of a number represented by a single figure, repeat the figure twice.

$11 \times 11 = 121$	$16 \times 11 = 176$
$12 \times 11 = 132$	$17 \times 11 = 187$
$13 \times 11 = 143$	$18 \times 11 = 198$
$14 \times 11 = 154$	$19 \times 11 = 209$
$15 \times 11 = 165$	$20 \times 11 = 220$

TWELVES

$1 \times 12 = 12$	$6 \times 12 = 72$	$11 \times 12 = 132$	$16 \times 12 = 192$
$2 \times 12 = 24$	$7 \times 12 = 84$	$12 \times 12 = 144$	$17 \times 12 = 204$
$3 \times 12 = 36$	$8 \times 12 = 96$	$13 \times 12 = 156$	$18 \times 12 = 216$
$4 \times 12 = 48$	$9 \times 12 = 108$	$14 \times 12 = 168$	$19 \times 12 = 228$
$5 \times 12 = 60$	$10 \times 12 = 120$	$15 \times 12 = 180$	$20 \times 12 = 240$

QUESTIONS

1. $12 \times 11 = ?$ 2. $16 \times 11 = ?$ 3. $19 \times 11 = ?$

Notice that any multiple by 11, of a number from 11 to 18, is represented by three figures, of which the hundreds' figure is that of the tens of the multiplicand, the tens' figure represents the sum of the two figures of the multiplicand, and the units' figure is the units' figure of the multiplicand.

Sometimes noticing these curiosities of arithmetic makes the study more interesting.

GENERAL MULTIPLICATION TABLE

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	4	6	8								
3	6	9	12								
4	8	12	16								
5					30			45			
6				30							72
7							56				
8						56	64				
9			36						90		
10								90			
11										121	
12					72						144

1. On a sheet of paper mark off 144 half-inch squares. Copy very accurately the numbers here. Fill in each blank square by the multiple of the numbers at the head of the column and at the left end of the row. $4 \times 2 = 8$ and $2 \times 4 = 8$. $11 \times 11 = 121$ and $12 \times 12 = 144$.

2. Compare your results with the multiplication tables in this book.

3. On the blackboard make 144 two-inch squares and proceed as in **1**.

4. Why are the numbers larger, the nearer they are to the lower right hand corner of the *table?*

MULTIPLYING BY 10; 100; 1000

1. $562 \times 10 = ?$

$$\begin{array}{r} 562 \\ \underline{10} \\ 5620 \end{array}$$

2. $3894 \times 10 = ?$

$$\begin{array}{r} 3894 \\ \underline{10} \\ 38940 \end{array}$$

To multiply by 10, add a zero at the right.

3. $\$562.25 \times 10 = ?$

$$\begin{array}{r} \$562.25 \\ \underline{10} \\ \$5622.50 \end{array}$$

4. $\$3894.40 \times 10 = ?$

$$\begin{array}{r} \$3894.40 \\ \underline{10} \\ \$38944.00 \end{array}$$

To multiply by 10 a sum of money expressed in dollars and cents, add a zero at the right and move the decimal point one place to the right.

5. $8729 \times 10 = ?$ **6.** $45{,}287 \times 10 = ?$ **7.** $900 \times 10 = ?$

8. $\$432.50 \times 10 = ?$ **9.** $\$92.75 \times 10 = ?$

10. $349 \times 100 = ?$

$$\begin{array}{r} 349 \\ \underline{100} \\ 34900 \end{array}$$

11. $4876 \times 1000 = ?$

$$\begin{array}{r} 4876 \\ \underline{1000} \\ 4876000 \end{array}$$

To multiply by 100, add two zeros at the right.

To multiply by 10, 100, 1000, 10,000, 100,000, 1,000,000, etc., add as many zeros at the right as the multiplier contains zeros.

12. Multiply 472, 8349, 5461, 289, and 4567 by 100, 1000, 10,000 and 100,000.

To multiply by 10, 100, 1000, 10,000, 100,000, 1,000,000, sums of money expressed in dollars and cents, move the decimal point in the product as many places farther to the right as there are zeros in the multiplier.

QUESTIONS

1. Multiply $329 by 1000, and divide the product by 7.

2. Write the Multiplication Table of Elevens.

3. What is $\frac{1}{8}$ of 64, 96, 80, 32, 24, 48, 88, 72, 56?

4. What is the ratio to 12 of 108, 84, 144, 60?

5. Draw a plan for a room 16 ft. × 12 ft. to the scale of $\frac{1}{4}$ in. to 1 ft.

6. A man's property at his death was found to be worth $90,000. He left a wife and five children. To the widow he gave $\frac{1}{3}$ of the property, and to each of his children $\frac{1}{5}$ of the remainder. How much did each receive?

7. Which is greater, one hundred forty million or one thousand times ten thousand? What is the ratio of the first number to the product of the other numbers?

8. If one thousand men earn by work $2 a day each, and another man effects a saving in a great business so that he receives for his work $2000 a week, what is the ratio of the weekly earnings of all the thousand men for six days to the salary of the one man?

9. Is it more to save $10 a month or to save $3 a week? Why?

10. There are 800 pages in one book which costs $4, and 300 pages in another book which costs $2. If the pages of each book are equally valuable, which book is cheaper?

11. One man bought 2 doz. fresh eggs for 40¢, another man bought $\frac{1}{2}$ doz. for 12¢. Which eggs were cheaper?

12. A boy wore out 4 pr. of $2 shoes a year, another boy wore out 6 pr. of $1.50 shoes, and still another boy wore out 3 pr. of $3 shoes. Which boy's shoes cost his father the most money a year, if the cost of repairs is not counted?

MULTIPLICATION

1. Multiply 15,468 by 627.

	15468	. . . .	multiplicand
	627	. . . .	multiplier
7 × 15468 =	108276	. . . .	first partial product
20 × 15468 =	30936	. . . .	second partial product
600 × 15468 =	92808	. . . .	third partial product
627 × 15468 =	9698436	. . . .	total product

2. Multiply 29,546 by 703.

	29546	. . . .	multiplicand
	703	. . . .	multiplier
3 × 29546 =	88638	. . . .	first partial product
700 × 29546 =	206822	. . . .	third partial product
703 × 29546 =	20770838	. . . .	total product

3.
```
   46227
    8006
  277362
369816
370093362
```

4.
```
   88743
    5600
53245800
443715
496960800
```

Since we cannot multiply by zeros, which indicate absence of value in the places they occupy, we set down the zeros to show where the next product of the multiplication is to begin.

5. Mary's father bought 640 acres of land for $175 an acre. What was the cost of the section of land?

6. The government employed 34,529 private soldiers at a cost in board and wages of $31.90 a month each on the average. In one year what was the total cost of the private soldiers for these two expenses?

7. Multiply 43,287 by 192, 147, 298, 3753, and 4665.

MULTIPLICATION

Multiplication repeats one number as many times as there are units in another.

The number repeated, or multiplied, is the **multiplicand.**

The number showing how many times the multiplicand is repeated is the **multiplier.**

The result of the multiplication is the **product.**

The sign of multiplication, $\times$, is read **multiplied by.** It is also read *times.*

$75 \times 5 = ?$ may be read: Five times seventy-five, or Seventy-five multiplied by five, is what?

$1896 \times 245 = ?$ We may say either: Find 245 times 1896; or, Multiply 1896 by 245. It is usually better to read, Multiply.

Multiply:

1. 7019 by 203, 405, 607, 809, 506, 705.
2. 45,608 by 314, 415, 516, 617, 819, 218.
3. 45,609 by 340, 450, 560, 670, 780, 890.
4. 317,018 by 234, 345, 456, 567, 789, 895.
5. 49,017 by 451, 671, 891, 341, 751, 861.
6. 68,907 by 237, 459, 738, 426, 954, 765.
7. There were one hundred nine schools in the city of Brighton, and the average number of pupils in each school was two hundred ninety-eight. How many pupils were there in all?
8. There were nine hundred forty-six houses in the town of Walton, and the average number of panes of glass in each house was one hundred eighty-three. How many panes of glass were there in all?
9. If in each house two panes of glass were broken in each year, how many new panes in all were needed?

MULTIPLICATION

When we have multiplied two numbers together, one way to find whether our product is probably correct is to use the multiplier as multiplicand, and to multiply the numbers again.

652	Multiplicand	Proof:	314
314	Multiplier		652
2608			628
652			1570
1956			1884
204728	Product		204728

Multiply and prove in this way:

1. 48	2. 97	3. 149	4. 556	5. 1242	6. 2987
36	15	167	497	1298	260

7. Multiply 3291 by 8272. Prove by exchanging multiplier and multiplicand.

8. Multiply 4875 by 9291; and prove the answer.

9. In a great battle there were engaged on both sides together two hundred sixteen regiments with an average of six hundred fifty soldiers in each regiment. How many soldiers were engaged in all?

10. If one fifth of all the soldiers engaged were killed or wounded or deserted to go home, how many were left in both armies?

11. If in 1900 there were about 84,000,000 in all the United States and its possessions, and if 16,000,000 of these were colored people, what was the ratio of the white people to the colored people?

GENERAL MULTIPLICATION TABLE

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

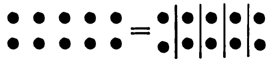
1. Read the multiplication table of each number, beginning $2 \times 1 = 2$, $2 \times 2 = 4$, $2 \times 3 = 6$, and so through 2's; then $3 \times 1 = 3$, and so on through all numbers.

2. Read the division facts in this way, beginning $4 \div 2 = 2$, $6 \div 2 = 3$, $8 \div 2 = 4$, and so through the first column; then $6 \div 3 = 2$, $9 \div 3 = 3$, $12 \div 3 = 4$; and so on through all the numbers.

3. Read the columns down, 2, 4, 6, 8, and so on; 3, 6, 9, 12, and so on, telling in what multiplication table we find these numbers.

4. What numbers multiplied together give 144, 132, 121, 120, 110, 108, 100, 99, and so on through all these numbers?

DIVISION

If we divide one number by another, we find how many times the divisor is in the dividend. Let us divide 10 into 2's:  = 5 2's.

Dividing is separating into parts. When we divide 48 by 12, we find 4 parts of 12 units each.

$12+12+12+12=48$

$48-12-12-12-12=0$

$$\begin{array}{r} 12 \\ 12 \\ 12 \\ \underline{12} \\ 48 \end{array}$$

It is much easier to multiply two numbers than to add many. Division is also easier than many subtractions.

If we wished to separate 360 into parts of 10 each, how hard it would be to find the answer in this way: $360-10=350$, $350-10=340$, $340-10=330$, etc.; and then to count all the 10's! Instead, we can write $\begin{array}{r} 10\overline{)360} \\ \hline 36 \end{array}$. There are 36 10's in 360.

DIVIDING BY 10, 100, 1000

1. Divide 8000 by 100.

$1\cancel{0}\cancel{0})80\cancel{0}\cancel{0}$ Cancel two zeros in 100 and 8000.

$\underline{80}$ $8000 = 80$ 100's.

2. Divide six millions by three millions.

Cancel the millions. $\frac{6}{3} = 2$.

3. If New York City had in 1900 about $3\frac{1}{2}$ millions of people, and the whole United States had in 1800 7 millions, what was the ratio of population in all the United States then to New York one century later?

Divide: **4.** 64,000 by 8000. **5.** 15,000,000 by 3,000,000.

ADDITION

Add by columns and by rows:

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
A	2	9	5	1	4	3	8	7	2	1	2	4
B	2	8	5	2	4	3	8	8	1	3	3	5
C	3	8	5	3	4	3	4	9	1	5	4	6
D	7	6	5	4	4	3	5	9	1	8	4	5
E	7	6	3	5	7	2	6	8	1	9	4	4
F	5	2	9	9	8	2	7	7	3	1	3	6
G	4	9	7	8	8	2	8	5	3	9	2	4
H	6	8	4	8	7	9	9	4	5	7	5	9
I	6	7	4	8	9	9	2	2	5	8	4	6
J	6	7	5	8	6	9	1	6	1	9	1	2
K	1	5	9	9	1	5	1	4	6	7	4	8
L	9	1	8	6	8	5	3	3	2	8	5	7

SUBTRACTION

10	9	8	6	8	10	9	7	10	9	5	6
4	3	5	2	4	7	4	2	8	7	3	3

MULTIPLICATION

Multiply 2685 by 73, 82, 45, 19, 64, 28, and 57.

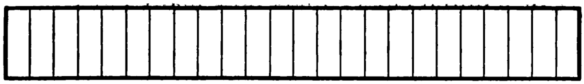
Multiply 4193 by the same numbers above.

DIVISION

Divide 4369 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

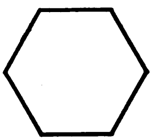
Divide 37,841 by 15, 16, 17, 18, 19, 25, 33, 47, and 58.

FRACTIONS

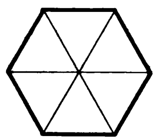


1. Point out $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{24}$ of this oblong.
2. Show that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{6}{12} = \frac{12}{24}$.
3. Show that $\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{8}{24}$.

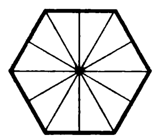
Hexagon



Sixths

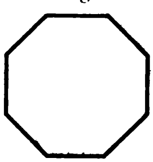


Twelfths

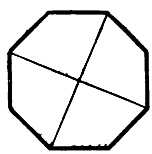


4. Show that $\frac{1}{6} = \frac{2}{12}$, $\frac{2}{6} = \frac{4}{12}$, $\frac{3}{6} = \frac{6}{12}$.

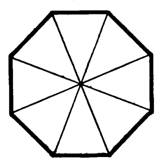
Octagon



Fourths

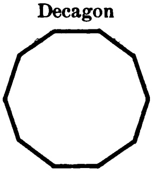


Eighths

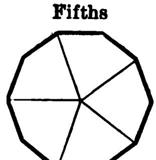


5. Show that $\frac{1}{4} = \frac{2}{8}$, $\frac{2}{4} = \frac{4}{8}$, $\frac{3}{4} = \frac{6}{8}$.

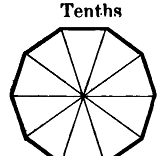
Decagon



Fifths



Tenths



6. Show that $\frac{1}{5} = \frac{2}{10}$, $\frac{2}{5} = \frac{4}{10}$, $\frac{3}{5} = \frac{6}{10}$.

FRACTIONS

1. $\frac{1}{6}$ of 2 dozen eggs equals how many eggs?

2. $\frac{1}{8}$ of 24 eggs equals how many eggs?

3. Which is the larger fraction, one sixth or one eighth of the same thing?

4. $\frac{3}{6}$ of 2 dozen eggs are how many eggs?

5. How many eggs are there in $\frac{4}{8}$ of 2 dozen eggs?

6. James is 16 years old, and his brother Willie is $\frac{1}{4}$ as old. How old is Willie?

7. How many thirds of a dollar are there in two dollars? in three dollars? in four dollars?

8. $\frac{1}{2} = \frac{?}{4}$ $\frac{1}{2} = \frac{?}{6}$ $\frac{1}{2} = \frac{?}{8}$.

9. $\frac{1}{5} = \frac{?}{10}$ $\frac{3}{5} = \frac{?}{10}$ $\frac{5}{5} = \frac{?}{10}$.

10. $\frac{1}{7}$ of 21 cents = how many cents? $\frac{2}{7}$? $\frac{3}{7}$? $\frac{7}{7}$?

11. $\frac{1}{9}$ of 27 cents = how many cents? $\frac{2}{9}$? $\frac{4}{9}$? $\frac{7}{9}$?

12. $\frac{1}{3}$ of 27 cents = how many cents? $\frac{2}{3}$?

13. If $\frac{2}{7}$ of a boy's money equal 14¢, how much does $\frac{1}{7}$ equal?

14. How many sixths are there in an apple? How many sixths of an apple are there in an apple and a half? How many eighths are there in two oranges?

15. Since $\frac{1}{3} = \frac{3}{9}$, how many thirds $= \frac{6}{9}$?

16. How many oranges are there in $\frac{1}{2}$ of a dozen?

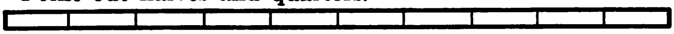
17. How many oranges are there in $\frac{3}{4}$ of a dozen?

18. If you had $\frac{7}{7}$ of anything, would you have $\frac{3}{3}$ of it?

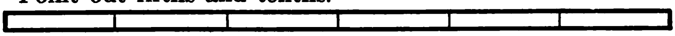
19. If $\frac{1}{8}$ of a pound of butter is worth 3¢, how much is a pound worth?

FRACTIONS

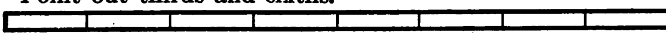
Point out halves and quarters.



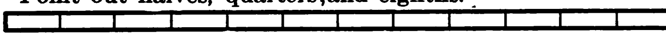
Point out fifths and tenths.



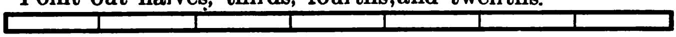
Point out thirds and sixths.



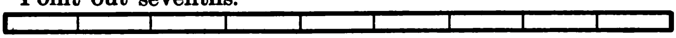
Point out halves, quarters, and eighths.



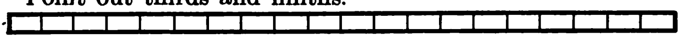
Point out halves, thirds, fourths, and twelfths.



Point out sevenths.



Point out thirds and ninths.



Point out twentieths, tenths, and fifths.

EXERCISES

1. Draw a one-inch line. Add $\frac{1}{2}$ to it; $\frac{1}{4}$; $\frac{1}{8}$. How long is the whole line?

2. Draw a two-inch line. Add $\frac{1}{4}$ to it; $\frac{1}{8}$; $\frac{1}{16}$. How long is the whole line?

3. Draw a three-inch line. Add $\frac{1}{3}$ to it; $\frac{1}{6}$; $\frac{1}{9}$; $\frac{1}{18}$. How long is the whole line?

4. Draw a five-inch line. Add $\frac{1}{5}$ to it; $\frac{1}{10}$; $\frac{1}{20}$; $\frac{1}{40}$. How long is the whole line?

5. Divide the five-inch line by 2. How long is each part?

6. Draw a six-inch line. Divide each inch into $\frac{1}{8}$ inches. How many $\frac{1}{8}$ inches are there in all?

7. Explain what you have done in each case.

THE FRACTION

A **unit** is one, a whole thing.

A **fraction** is one or more of the equal parts of a unit.

A **common fraction** is expressed by two numbers, one number written below the other, with a line between them. Thus $\frac{7}{8}$, $\frac{9}{10}$, $\frac{1}{2}$, are common fractions.

The **terms** of a fraction are the numerator and the denominator.

The number below the line is called the **denominator.** It tells into how many equal parts the unit has been divided. The greater the number of parts, the smaller is each part. $\frac{3}{8}$ is less than $\frac{3}{7}$. $\frac{12}{20}$ is greater than $\frac{12}{40}$. $\frac{1}{4}$ of an orange is more than $\frac{1}{6}$ of the orange. Show by drawings.

The number above the line is called the **numerator.** It tells the number of equal parts taken.

$$\begin{array}{rc} \text{numerator} & 5 \\ \hline \text{denominator} & 6 \end{array}$$

A **proper fraction** is a fraction whose value is less than one. Its numerator is less than its denominator; $\frac{2}{3}$, $\frac{7}{9}$, $\frac{13}{14}$, $\frac{91}{97}$.

An **improper fraction** is a fraction whose value is equal to or greater than one. Its numerator is equal to or greater than its denominator; $\frac{4}{4}$, $\frac{7}{7}$, $\frac{12}{8}$, $\frac{21}{10}$.

A **mixed number** is a number consisting of a whole number and a fraction; $7\frac{1}{6}$, $9\frac{3}{4}$, $31\frac{5}{7}$.

A fraction is an expression of division. Thus **twelve divided by three** may be written $3\overline{)12}$, or $12 \div 3$, or $\frac{12}{3}$.

Tell the proper fractions, the improper fractions, and the mixed numbers: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{8}$, $\frac{9}{3}$, $2\frac{1}{3}$, $\frac{8}{9}$, $6\frac{3}{4}$, $\frac{12}{4}$, $7\frac{1}{2}$, $\frac{5}{5}$, $\frac{16}{8}$, $\frac{3}{3}$, $\frac{20}{5}$, $\frac{6}{9}$, $20\frac{1}{6}$, $\frac{10}{2}$, $\frac{2}{1}$, $31\frac{1}{9}$, $\frac{9}{9}$, $1\frac{1}{3}$, $\frac{17}{18}$, $\frac{25}{5}$, $\frac{21}{3}$, $\frac{3}{20}$, $14\frac{8}{9}$, $\frac{30}{30}$, $5\frac{4}{5}$, $\frac{22}{11}$, $9\frac{4}{5}$, $\frac{1}{8}$, $\frac{16}{8}$, $\frac{20}{4}$.

UNITS ADDITION

Notice, when adding columns, all combinations making:

(1) 5, 10, 15, or 20; or (2) 9; or (3) 11; or (4) two or three times the same numbers, as 3, 3, 3; or 4, 4, 4.

A

$$\begin{array}{rr} \left.\begin{matrix}2\\3\end{matrix}\right\} & 5 \\ \left.\begin{matrix}6\\4\end{matrix}\right\} & 10 \\ \left.\begin{matrix}9\\6\end{matrix}\right\} & 15 \\ \left.\begin{matrix}7\\3\end{matrix}\right\} & 10 \\ \left.\begin{matrix}8\\7\end{matrix}\right\} & 15 \\ \left.\begin{matrix}8\\2\end{matrix}\right\} & 10 \\ \left.\begin{matrix}4\\1\end{matrix}\right\} & 5 \\ \hline 80 & 80 \end{array}$$

B

$$\begin{array}{rr} \left.\begin{matrix}1\\8\end{matrix}\right\} & 9 \\ \left.\begin{matrix}7\\2\end{matrix}\right\} & 9 \\ \left.\begin{matrix}6\\3\end{matrix}\right\} & 9 \\ \left.\begin{matrix}5\\4\end{matrix}\right\} & 9 \\ \hline 36 & 36 \end{array}$$

C

$$\begin{array}{rr} \left.\begin{matrix}2\\9\end{matrix}\right\} & 11 \\ \left.\begin{matrix}3\\8\end{matrix}\right\} & 11 \\ \left.\begin{matrix}4\\7\end{matrix}\right\} & 11 \\ \left.\begin{matrix}5\\6\end{matrix}\right\} & 11 \\ \hline 44 & 44 \end{array}$$

D_1

$$\begin{array}{rr} \left.\begin{matrix}3\\7\end{matrix}\right\} & 10 \\ \left.\begin{matrix}6\\2\\1\end{matrix}\right\} & 9 \\ \left.\begin{matrix}4\\7\end{matrix}\right\} & 11 \\ \left.\begin{matrix}2\\8\end{matrix}\right\} & 10 \\ 7 & 7 \\ \hline 47 & 47 \end{array}$$

Added from above.

D_2

$$\begin{array}{rr} 3 & 3 \\ \left.\begin{matrix}7\\6\\2\end{matrix}\right\} & 15 \\ \left.\begin{matrix}1\\4\end{matrix}\right\} & 5 \\ \left.\begin{matrix}7\\2\end{matrix}\right\} & 9 \\ \left.\begin{matrix}8\\7\end{matrix}\right\} & 15 \\ \hline 47 & 47 \end{array}$$

Added from below.

D_3

$$\begin{array}{rr} 3 & 3 \\ \left.\begin{matrix}7\\6\\2\\1\\4\end{matrix}\right\} & 20 \\ 7 & 7 \\ \left.\begin{matrix}2\\8\end{matrix}\right\} & 10 \\ 7 & 7 \\ \hline 47 & 47 \end{array}$$

Either

F

$$\begin{array}{rr} \left.\begin{matrix}4\\4\end{matrix}\right\} & 8 \\ 8 & 8 \\ \hline 16 & 16 \end{array}$$

G

$$\begin{array}{r} 7 \\ 7 \\ 7 \\ \hline 21 \end{array}$$

H

$$\begin{array}{rr} \left.\begin{matrix}2\\3\end{matrix}\right\} & 5 \\ 5 & 5 \\ \hline 10 & 10 \end{array}$$

Write these rows in columns and add:

I	5,	2,	3,	6,	9,	1,	7,	2,	1,	4,	3,	8,	7,	5,	8.
J	1,	9,	8,	7,	2,	4,	6,	1,	7,	6,	8,	8,	2,	1,	3.
K	4,	4,	4,	5,	5,	6,	3,	8,	7,	9,	6,	5,	9,	9,	2.
L	9,	9,	9,	6,	2,	5,	5,	6,	4,	3,	3,	3,	1,	7,	8.
M	3,	7,	6,	4,	5,	5,	8,	2,	9,	1,	4,	5,	1,	2,	5.

N Add the fifteen columns in I–M above.

REVIEW

1. A man had 315 horses; he sold 39, but bought 124 more horses. How many horses had he then?

2. In a gathering of 326 men, women, and children, there were 126 men and 150 women. How many children were there?

3. In a school there were 239 pupils, of whom 63 were girls. How many boys were there in the school?

4. Mr. Jones is 84 years old and his son is 59 years old. How much older is the father than the son?

5. Write, in words, the difference between ten thousand fifty and ninety-nine.

6. A coal dealer had 800 tons of coal, of which he sold 333 tons. How many tons had he left?

7. A farmer sold 114 sheep out of a flock of 313. How many sheep had he left?

8. Mr. Smith has 324 sheep and cows, while Mr. Brown has 275 sheep and cows. How many more animals has Mr. Smith than Mr. Brown?

9. A man starts to walk five hundred yards. When he has walked a hundred eighty-two yards, how many yards more has he to walk?

10. Five hundred thirteen children went on a steamboat trip. Three hundred five of the children were boys. How many girls were there?

11. If I have seventy-five dollars, and give away thirty dollars to one man and twenty-eight dollars to another man, how many dollars shall I have left?

12. In one schoolroom there are 34 children, in another room, 40 children, and in a third room, 29 children. Of 150 pupils enrolled how many pupils are absent?

GENERAL REVIEW

1. $\frac{1}{2}$ of 16 = ? $\frac{1}{4}$ of 32 = ? $\frac{1}{4}$ of 16 = ?

2. $\frac{1}{8}$ of 32 = ? How many eighths of 16 = $\frac{3}{8}$ of 32?

3. There are five trains, and each train has 23 cars. How many cars are there in the five trains?

4. How many gallons will 12 dozen bottles hold, when each bottle holds $\frac{1}{5}$ of a gallon?

5. $\frac{1}{8}$ of 80 = ? $\frac{3}{8}$ of 40 = ? $\frac{3}{8}$ of 80 = ?

6. $\frac{4}{8}$ of 40 = ? $\frac{6}{8}$ of 40 = ? $\frac{5}{8}$ of 40 = ?

7. How many eighths of 80 = $\frac{6}{8}$ of 40?

8. A number of pies were cut into 4 equal pieces each. How many pies would 28 of those pieces equal?

9. On Monday a horse went 23 miles, on Tuesday, 61 miles, on Wednesday and Thursday he rested, and on Friday he went 58 miles. How many miles did the horse travel in the five days?

10. $\frac{1}{8}$ of 64 = ? $\frac{2}{8}$ of 32 = ? $\frac{2}{8}$ of 64 = ?

11. How many eighths of 32 = $\frac{2}{8}$ of 64?

12. A merchant has 78 packages of baking soda. Each package weighs $\frac{1}{4}$ lb. How many pounds do all the packages weigh?

13. How many eighths of 32 = $\frac{1}{2}$ of 16?

14. How many fourths of 24 = $\frac{3}{8}$ of 48?

15. How many bushels are there in 729 baskets of plums, each basket containing $\frac{1}{5}$ bu.?

16. $\frac{1}{8}$ of 72 = ? $\frac{3}{8}$ of 24 = ? $\frac{5}{8}$ of 48 = ?

17. $\frac{2}{8}$ of 72 = ? How many eighths of 24 = $\frac{2}{7}$ of 72?

18. A farmer has 32 acres in one field, 41 acres in another field, 27 acres in a third field, and 9 acres in a fourth field. How many acres are there in all the fields?

STUDY 44

DIVISION

Division finds how many times one number is contained in another.

The number to be divided is called the **dividend.**

The number we divide by is called the **divisor.**

The result obtained by division is called the **quotient.** It shows how many times the divisor is contained in the dividend.

When the dividend does not contain the divisor an exact number of times, the part of the dividend left undivided is called the *remainder*, which is always less than the divisor.

The sign of division, ÷, shows that the number before it is to be divided by the number after it. Thus,

$10 \div 5 = 2$. Ten divided by five is two.

Division is also indicated by writing the dividend above a line and the divisor below it; thus, $\frac{10}{5} = 2$.

The sign) also is used to indicate division. Thus 5)10(2 shows that 10 divided by 5 equals 2. We sometimes show by this form, $8\underline{)64}$, that division is desired.

Proof. Multiply the quotient by the divisor and add the remainder, if any. If the result equals the dividend, the work is correct.

$$\begin{array}{l} 8\underline{)65} \\ \quad 8+1 \end{array} \qquad 8 \times 8 = 64 + 1 \qquad \begin{array}{r} 8\underline{)216} \\ 27 \end{array} \qquad \begin{array}{rl} 27 & \text{quotient} \\ \times 8 & \text{divisor} \\ \hline 216 & \text{dividend} \end{array}$$

Divide 433 by (1) 5; (2) 7; (3) 19.

By introducing the fraction we may secure exact division always. To do this we treat the remainder as a fraction of which the divisor is the denominator and the remainder the numerator.

$$\begin{array}{l} 8\underline{)65} \\ \quad 8\frac{1}{8} \end{array} \quad 8 \times 8\tfrac{1}{8} = (8 \times 8) + (8 \times \tfrac{1}{8}) = 64 + 1 = 65.$$

SHORT DIVISION

We know now the multiplication tables of all numbers from 2 to 12. We call division by these numbers **short division** because there is very little writing to do.

$$
\begin{array}{r}
8\underline{)89616} \\
11202
\end{array}
\qquad
\begin{array}{r}
8\underline{)192168} \\
24021
\end{array}
\qquad
\begin{array}{r}
9\underline{)90189} \\
10021
\end{array}
\qquad
\begin{array}{r}
7\underline{)1109964} \\
158566\tfrac{2}{7}
\end{array}
$$

The first of these problems would be in **long division** if we put down all the processes in full.

$$
\begin{array}{l l}
8)89616(11202 & \\
\underline{8} & 1 \times 8 = 8 \\
09 & \\
\underline{08} & 1 \times 8 = 8 \\
16 & \\
\underline{16} & 2 \times 8 = 16 \\
01 & \\
\underline{00} & 0 \times 8 = 0 \\
16 & \\
16 & 2 \times 8 = 16
\end{array}
$$

Divide each dividend in each group by each divisor.

GROUP I				GROUP II			
	Dividend	**Divisor**			**Dividend**	**Divisor**	
1.	11,680	(*a*)	2	**5.**	732,119	(*e*)	6
2.	75,010	(*b*)	3	**6.**	732,107	(*f*)	7
3.	80,171	(*c*)	4	**7.**	11,988,959	(*g*)	8
4.	110,519	(*d*)	5	**8.**	11,010,989	(*h*)	9

	Dividend	**Divisors**
9–10–11–12.	510,384 ÷	7, 8, 9, 11 =
13–14–15–16.	3,627,089 ÷	6, 7, 9, 12 =
17–18–19–20.	1,084,608 ÷	12, 9, 6, 3 =
21–22–23–24.	1,018,193 ÷	11, 9, 7, 5 =

LONG DIVISION

1. Divide 4365 by 14.

```
14)4365(311 11/14 . . . . quotient
   42
   ---
    16
    14
    ---
     25
     14
     ---
     11
```

14 is contained in 43 hundreds 3 (hundreds) times and 1 hundred remaining. We write 3 in hundreds' place in the quotient. 1 hundred and 6 tens = 16 tens. 14 is contained in 16 tens 1 (ten) times and 2 (tens) remaining. We write 1 in tens' place. 2 tens and 5 units = 25 units. 14 is contained in 25 units 1 (unit) time and 11 units remaining. We write 1 in units' place and put 11 over 14, with a line between the two figures to show that the 11 is still to be divided by 14.

The process of **long division** is the same as that of short division, but in the former the work is written out in full. When the divisor is larger than 12, we generally employ the method of **long division.**

2. Divide 17,092 by 23.

```
23)17092(743 3/23
   161
   ---
     99
     92
     ---
      72
      69
      ---
       3
```

```
Proof:  743 . . quotient
         23 . . divisor
       ----
       2229
      1486
          3 . . remainder
      -----
      17092 . . dividend
```

3. Divide 96,875 by 57.
4. Divide 105,290 by 75.
5. Divide 41,278 by 24.
6. Divide 86,397 by 17.
7. Divide 192,689 by 33.
8. Divide 207,175 by 25.
9. 66)432,824(
10. 59)127,567(
11. 37)368,129(
12. $82932 \div 24 = ?$
13. $96528 \div 99 = ?$
14. $125625 \div 25 = ?$

LONG DIVISION

Group I

Divide	I.	27573	(*a*)	13
each	II.	3378	(*b*)	16
dividend	III.	9034	(*c*)	17
by each	IV.	3978	(*d*)	19
divisor	V.	7605	(*e*)	18

Group II

Divide	VI.	910,372	(*f*)	93
each	VII.	588,168	(*g*)	84
dividend	VIII.	11,101	(*h*)	101
by each	IX.	98,200	(*i*)	99
divisor	X.	64,466	(*j*)	94

Why will there be twenty-five different quotients in each of these three groups?

Group III

Divide	XI.	92,483	(*k*)	23
each	XII.	78,104	(*l*)	26
dividend	XIII.	13,527	(*m*)	27
by each	XIV.	13,066	(*n*)	65
divisor	XV.	32,216	(*o*)	46

1. An excursion steamer is licensed to carry 425 passengers. How many trips must it make to carry 3825?

2. A boat licensed to take no more than 125 passengers a trip has 2337 persons to transport. How many trips must it make to do so? If it takes 125 passengers on each trip but the last one, how many passengers will it carry on the last trip?

3. A man left property valued at six hundred thousand dollars to be divided equally among his eleven children, after giving his wife one third of all the property. How many dollars should be given to each heir?

REVIEW

1. At $3 a dozen, how many chickens can be bought for $15.75?

2. A man bought 6 house lots for $5580. What was the average cost of each lot?

3. When land is worth $76 an acre, how many acres can one buy with $12,312?

4. A person's income is $1568 a year. What is his income a week?

5. A newsboy earns $19 a month. How many years will it take him to earn $1368?

6. If one door knob takes half a pound of metal, how many door knobs will 2528 ounces of metal make?

7. There are 196 pounds of flour in a barrel. How many barrels can be packed from 18,816 pounds of flour?

8. If the yield of 110 sugar canes is 6 pounds of sugar, how many pounds of sugar will 1,233,210 canes yield?

9. In a regiment of 1280 men there were 64 officers. How many privates were there to each officer?

10. In how many days will a traveler, driving at the rate of six miles an hour for seven hours a day, complete a journey of five hundred miles?

11. For one page of a book containing 36 lines, 2052 letters were used. How many letters, on an average, were there in each line?

12. If you had to divide 72 dozen oranges among 430 children, how many whole oranges could you give to each child? How many oranges would you have left?

13. The circumference of the globe at the equator is about 24,902 miles. How many hours would it take a train, traveling 28 miles per hour, to go around the globe?

14. Draw a hexagon. Divide it into six triangles.

ADDITION PROBLEMS

1. How many men were there in an army composed of 75,600 infantry, 25,087 cavalry, 14,097 artillery, 6095 men in the engineer corps, 1095 men in the medical corps, and 1278 in the commissary department?

2. How many bricks are there in seven different heaps of 7069, 25,804, 716, 4684, 97,605, 120,176, and nine thousand seventy bricks each?

3. A wholesale grocery firm had in stock seven thousand ninety-six pounds of rice, twenty thousand four pounds of sugar, 918 pounds of pepper, 4079 pounds of cocoa. What was the total weight of all these goods?

4. An army had ten thousand men in the ranks, 2019 sick, 1409 home on furlough, 4060 had been killed and wounded, 619 were prisoners. How many men were in the army at first?

5. There were brought into the port of Boston, in one day, eight hundred thousand nine herring, 7016 bluefish, 508 haddock, 20,016 cod, 1219 shad, 619 hake, and nineteen hundred mackerel. How many fish in all were brought in?

6. A coal district yielded 10,609 tons on Monday, 7018 tons on Tuesday, 20,109 tons on Wednesday, 9600 tons on Thursday, 14,700 tons on Friday, 16,849 tons on Saturday. What was the weekly output of coal?

7. In one year, San Francisco sent 49,684 barrels of flour to Asiatic ports, 29,344 barrels to South American ports, 10,968 barrels to islands in the Pacific, and 390,897 barrels to other parts of the globe. How many barrels in all were sent away?

8. Add: \$10.07, 90¢, \$8.92, \$10, \$5.01.

REVIEW

Divide:

1. 463,750 by 7, 8, 9, 11.
2. 4,200,560 by 5, 7, 8, 10.
3. 7,200,045 by 9, 8, 7, 6.
4. 4,704,091 by 7, 11, 9, 5.
5. 94,328 by 6, 8, 9, 10.
6. 43,272 by 3, 6, 9, 12.
7. 377,424 by 4, 6, 7, 8.
8. 975,216 by 5, 6, 8, 9.

Multiply:

A. 5019 by 14, 15, 18, 21, 27, 32, 36, 42.
B. 67,895 by 56, 63, 72, 84, 96, 77, 45, 54.
C. 75,098 by 16, 20, 30, 40, 50, 60, 70, 80.
D. 97,014 by 27, 72, 63, 36, 54, 45, 88, 99.
E. 80,694 by 96, 60, 48, 84, 121, 132, 144.
F. 97,065 by 132, 121, 108, 99, 120, 100, 110.

I. In the problems 1 to 8 in division, prove the quotients correct by multiplying them by the divisors, and seeing if you get the dividends.

II. In the problems A to F in multiplication, prove the products correct by dividing them by the multipliers, and seeing if you get the multiplicands.

III. Do you see now the value of knowing thoroughly the multiplication tables? Are they as useful in division as in multiplication? Why?

PUNCTUATION

If it helps you to be accurate, write 463,750 with the comma, to show thousands, and 4,200,560 with the commas, to make the thousands and millions plain.

MAKING PROBLEMS

Using such numbers as these on this page, make oral or written problems like those in different parts of this book.

ALIQUOT PARTS OF 100

1. $\frac{100}{3} = 33\frac{1}{3}$ $33\frac{1}{3}$, $66\frac{2}{3}$, 100 $33\frac{1}{3} \times 3 = 100$.
2. $\frac{100}{6} = 16\frac{2}{3}$ $16\frac{2}{3}$, $33\frac{1}{3}$, 50, $66\frac{2}{3}$, $83\frac{1}{3}$, 100.
3. $\frac{100}{8} = 12\frac{1}{2}$ $12\frac{1}{2}$, 25, $37\frac{1}{2}$, 50, $62\frac{1}{2}$, 75, $87\frac{1}{2}$, 100.
4. $\frac{100}{12} = 8\frac{1}{3}$ $8\frac{1}{3}$, $16\frac{2}{3}$, 25, $33\frac{1}{3}$, $41\frac{2}{3}$, 50.
5. $\frac{100}{16} = 6\frac{1}{4}$ $6\frac{1}{4}$, $12\frac{1}{2}$, $18\frac{3}{4}$, 25, $31\frac{1}{4}$, $37\frac{1}{2}$, $43\frac{3}{4}$, 50.
6. $16\frac{2}{3} \times 6 = ?$ $12\frac{1}{2} \times 8 = ?$ $8\frac{1}{3} \times 12 = ?$ $6\frac{1}{4} \times 16 = ?$
7. What is the ratio of $\frac{1}{6}$ of 100 to $\frac{1}{12}$ of 100?
8. What is the ratio of $\frac{1}{16}$ of 100 to $\frac{1}{8}$ of 100?
9. What part of 100 is $37\frac{1}{2}$? $87\frac{1}{2}$? 75?
10. What part of 100 is $16\frac{2}{3}$? $41\frac{2}{3}$? $83\frac{1}{3}$?
11. What part of 100 is $6\frac{1}{4}$? $31\frac{1}{4}$? $37\frac{1}{2}$?

12. John bought 100 marbles. He gave $\frac{1}{5}$ to his brother Charles, $\frac{1}{10}$ to his brother Will, and $\frac{1}{4}$ to his brother Thomas. How many had he left?

13. Mary had one dollar and bought two yards of wide silk ribbon at $37\frac{1}{2}$¢ per yard. What part of a dollar should she receive in change?

REVIEW

WRITE

14. Find the population of a town which has 16,009 girls, 15,700 boys, 35,019 women, and 37,009 men.

15. In 6729 eggs are how many dozen?

16. Seven thousand bushels of wheat, 16,009 of barley, nine thousand twelve of oats, 6019 of beans, 14,076 of peas, 8019 of rye, and 756 of buckwheat, are how many bushels in all?

PERCENTAGE

How many cents are there in a dollar? What part of a dollar is one cent?

3¢ are how many hundredths of a dollar?

6¢ are how many hundredths of a dollar?

7 hundredths of a dollar are how many cents?

1 hundredth of a dollar is one per cent of it.

2 hundredths of any number is two per cent of it.

4 hundredths of any number is four per cent of it.

27 per cent of a number is 27 hundredths of it.

Any per cent of a number is an equal number of hundredths of that number.

In business the sign % is used for *per cent.*

Per cent or % may be read *hundredths.*

3 per cent = 3%;	6 per cent = 6%;
10 per cent = 10%;	45 per cent = 45%;
91 per cent = 91%;	100 per cent = 100%.

Any rate per cent, being a rate per hundred, may be written as a common fraction or a decimal fraction.

$$1 \text{ per cent} = \tfrac{1}{100} = 1\% = .01.$$
$$7 \text{ per cent} = \tfrac{7}{100} = 7\% = .07.$$
$$34 \text{ per cent} = \tfrac{34}{100} = 34\% = .34.$$

What is $16\frac{2}{3}\%$ of 96 bu.? $16\frac{2}{3} \times 6 = 100.$

$$6\overline{)96}$$
$$16$$

$16\frac{2}{3}\%$ of any number is $\frac{1}{6}$ of the number.

Dividing 96 by 6 we obtain 16 bu., the required percentage of 96 bu.

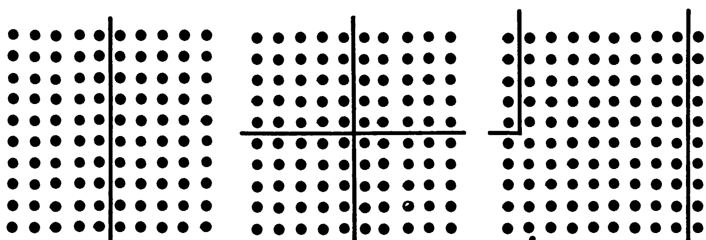
USE OF PERCENTAGE

Per cents are especially used in dealing with money; for when men lend money to other men, the lenders usually ask the borrowers not only to give them back after a time all their money, but also to pay them so much per cent for the use of the money. This payment is called **interest.** Also we pay the governments of our town or city and of our state every year so much per cent of the money value of our property. This payment is called a **tax.** It supports the police and schools and takes care of the roads.

4% of \$1000 is $\frac{4}{100}$ of \$1000 $\quad$ $\frac{1}{100}$ of \$1000 = \$10

\$10 × 4 = \$40 $\quad$ 2% of \$5000 is $\frac{2}{100}$ of \$5000

$\frac{1}{100}$ of \$5000 = \$50 $\quad$ \$50 × 2 = \$100



50% $\quad$ 25% $\quad$ 5% $\quad$ 10%

1. What is the interest on \$300 for one year at 5%?

2. The tax on houses in Lawton is 2%. What amount must be paid on a house valued at \$4000?

3. In the same town what is the amount to be paid on a horse and carriage valued at \$200?

4. In Lawton what is the tax on an office building valued at \$100,000? on a factory valued at \$250,000?

SCHOOL PER CENTS

1. John had 87% in his arithmetic, 60% in reading, 80% in manual work, 75% in drawing, 70% in music, 60% in spelling, and 90% in Nature study. What was his average, if each study counted the same?

87%	
60	
80	
75	
70	
60	
90	
522	$7\overline{)522}$ = $74\frac{4}{7}\%$

2. But what was his per cent if arithmetic counted 3 points, reading and manual work 2 points each, and the other exercises 1 point each?

87%	×	3	=	261
60	×	2	=	120
80	×	2	=	160
75	×	1	=	75
70	×	1	=	70
60	×	1	=	60
90	×	1	=	90
		11		836

$11\overline{)836}$ = 76%

3. Find the averages of various reports.

4. In a class of 36 boys and girls $33\frac{1}{3}\%$ were girls. How many girls were there? How many boys? 50% were not absent in October. How many came every day? 25% were tardy once each during the entire school year. How many were tardy?

5. In another class of 45 boys and girls $33\frac{1}{3}\%$ were girls. How many boys were there? How many girls? 80% were not absent in October. How many came every day? 9 were tardy once during the year. What per cent of 45 was that? What is the ratio of 9 to 45? $\frac{1}{5}$ is what per cent of 100?

REVIEW

1. Divide:

a. 301,679 by 14, 15, 16, 18, 20, 22, 24, 28.
b. 750,643 by 21, 30, 32, 33, 36, 40, 42, 44.
c. 600,119 by 48, 50, 56, 60, 63, 64, 66, 70.
d. 412,065 by 72, 77, 80, 84, 88, 90, 96, 99.

2. How many oranges at 4¢ each can one buy for 56¢? For 72¢? For 80¢? For $1?

3. In a class at school 40% were boys and 60% were girls. There were in all 50 children. How many were boys? How many were girls?

4. What is the interest on $500 at 6% for one year?

5. Mary gave 10% of 70¢ to her sister Sarah. How many cents had she left?

6. What part of 100 is $16\frac{2}{3} \times 2$?

Answer this by answering $16\frac{2}{3}$ = what fraction of 100. Then multiply the fraction by 2.

7. Divide a rectangle into thirds and ninths.

8. How many lengths of 6 yards each can be cut from a piece of silk 42 yards long?

9. If 96 pounds of flour be made up into 32 loaves, how many pounds of flour will there be in each loaf?

10. A gentleman gave 91¢ to be divided equally among 7 boys. How many cents should each boy get?

11. Herbert was given 24¢ to be divided equally among himself and his three brothers. How many cents should he give to each of his brothers?

12. What is $12\frac{1}{2}$% of 32? 48? 96? 144?

13. Multiply 8275 by 26, 37, 18, and 45.

14. What is an **improper fraction?**

ADDITION

In addition, the numbers to be added are called **addends.**

Tell the **addends,** the **subtrahends,** the **minuends,** and **differences**; the **multiplicands,** the **multipliers,** and the **products**; the **dividends,** the **divisors,** and the **quotients**; the **ratios** and the **fractions** in the following questions:

1. A farmer bought 6 cows at $30 each from a trader, 4 cows at $32 each from a neighbor, and 2 cows at $75 each from a fancy stock farmer. He then had in all 15 cows. How many cows had he to begin with? How much did all the new cows cost him?

2. In a regiment of 708 men there were 120 veterans, and the rest were raw recruits. The average age of the veterans was 38 yr., while that of the recruits was 15 yr. less. How many raw recruits were there? What was their average age?

3. A boy bought two saws, a hammer, a hatchet, a file, a plane, a try-square, two chisels, a vise, and a 2-foot rule. These cost respectively $1.50 and $1 for the saws; 75¢, 90¢, 20¢, $1.35, 40¢, for the next tools; 40¢ and 30¢ for the chisels; $$2\frac{1}{4}$ and 30¢ for the last articles. How many tools did he buy? At what total cost? At the same general prices, how much would 33 tools have cost?

4. One girl had 7 paper dolls, another 3 times as many, another $\frac{1}{7}$ as many as the second, and a fourth as many as the second and third together. How many paper dolls had they altogether? If they could have sold them at 2¢ each, how much would each of them have received for her dolls? How much would all have received for all the dolls?

ROMAN NOTATION

This notation was first used by the ancient Romans. It employs seven capital letters.

In the prefaces of books small letters are used to mark the pages.

Our more common figures came from Arabia and India, lands even farther away than Rome in Italy.

I=1; V=5; X=10; L=50; C=100; D=500; M=1000.

TABLE

I=1	XX=20	LXXI=71	Compare these two ways of writing numbers and give some reasons why the Roman notation is easier and some reasons why it is harder than our common notation.
IV=4	XXI=21	LXXXI=81	
V=5	XXX=30	XC=90	
VI=6	XXXI=31	XCI=91	
IX=9	XL=40	CI=101	
X=10	XLI=41	CC=200	
XI=11	L=50	D=500	
XIV=14	LI=51	DC=600	
XV=15	LX=60	CM=900	
XVI=16	LXI=61	M=1000	
XIX=19	LXX=70	MM=2000	

PRINCIPLES

Repeating a letter repeats its value:

III, 3; XXX, 30; CC, 200; CCCXXII, 322.

When a letter is placed before one of greater value, the value of the less number is subtracted from the value of the greater; as, IV=5−1=4; XL=50−10=40.

XIX=10+10−1=19; XXIV=10+10+5−1=24.

When a letter is placed after one of greater value, the value of the less is added to the value of the greater; as,

VI=5+1=6; MC=1000+100=1100.

REVIEW QUESTIONS

1. Give the Multiplication Table of Twelves.

2. How can we at once write the product of any number by ten, one hundred, one thousand, etc.?

3. How many sides has a hexagon? a decagon? an octagon? Draw each on the blackboard?

4. What is the ratio of twelve apples to eighteen apples? to twenty-four? to six? to nine? to sixty?

5. Tell in the order of their sizes the following fractions: $\frac{1}{8}$ $\frac{1}{7}$ $\frac{1}{9}$ $\frac{1}{5}$ $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{6}$.

6. What does the denominator of a fraction tell? the numerator?

7. What kind of a fraction is $\frac{9}{7}$? $\frac{8}{7}$? $\frac{3}{5}$? $\frac{10}{4}$?

8. Change such fractions above as you think might be expressed as mixed numbers.

9. Add: 4, 3, 9, 8, 2, 6, 1, 3, 5, 8, 9, 4, 7, 4, 5.

10. What is the *quotient* in division? the *remainder*?

11. Give the aliquot parts of 100 divided in sixths.

12. Tell how we express 8% decimally.

13. A man borrowed $10,000 at 5% a year interest. How much interest did he owe at the end of one year on each $100? on each $1000? on the $10,000?

14. The tax on a house at 2% is $20. What part of the value of the house was this? How many times the tax was the house worth? What was the house worth?

15. What are *addends*?

16. An ancient building has upon its cornerstone the letters MCDXXIV. In what year was it built?

LONG MEASURE

Long measure is sometimes called **linear measure.** It is used in measuring length or distance.

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	= 1 rod (rd.)
320 rods or 5280 feet	= 1 mile (mi.)

1. How many inches are there in 2 ft.? 3 ft.? 7 ft.?

2. How many yards are there in 2 rd.? 6 rd.? 5 rd.?

3. How many rods are there in 2 miles? 3 miles? $3\frac{1}{2}$ miles? 100 miles?

4. How many inches are there in 1 yd.? 2 yd.? $1\frac{1}{2}$ yd.? $2\frac{1}{2}$ ft.? $2\frac{1}{4}$ yd.?

5. How many feet are there in 2 rd.? 4 rd.?

6. How many feet are there in 18 mi. 19 rd.?

$$\begin{array}{rl}
18 & \text{mi.} \\
\underline{320} & \\
360 & \\
\underline{54} & \\
5760 & \text{rd.} \\
\underline{19} & \text{rd.} \\
5779 & \text{rd.} \\
\underline{16\frac{1}{2}} & \\
2889\frac{1}{2} & \\
34674\phantom{\frac{1}{2}} & \\
\underline{5779\phantom{\frac{1}{2}}} & \\
95353\frac{1}{2} & \text{ft.}
\end{array}$$

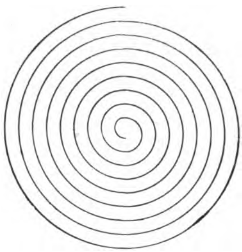
As there are 320 rd. in a mile, in 18 miles there are 320 times 18 rd., or 5760 rd. Adding the 19 rd., we obtain 5779 rd. As there are $16\frac{1}{2}$ ft. in 1 rd., in 5779 rd. there are $16\frac{1}{2}$ times 5779 ft., or $95{,}353\frac{1}{2}$ ft. Hence in 18 miles 19 rd. there are 95,353 ft.

In multiplying by $16\frac{1}{2}$, it is convenient to take $\frac{1}{2}$ of the multiplicand first.

$$2\overline{)5779}$$
$$2889\tfrac{1}{2}$$

To this we add the product of 5779 by the whole number, 16.

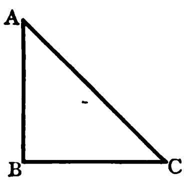
LENGTHS



Not all things to be measured are in straight lines.

A clock spring, 18 inches long, would be coiled up like this. Coil up 3 yards of thread or string, and see how it looks.

RIGHT ANGLE TRIANGLES



If we wish to cross a street and to save time and distance, we cut corners. If we wish to go to C from A, we may go from A to B and from B to C. Make drawings of triangles and measure.

The lines AB, BC, and CA form the **perimeter** of the triangle ABC.

BC is the **base.** AB is the **height.**

AC is the **hypotenuse.** The angle ABC is a **right** angle.

The angles CAB and ACB are **acute** angles.

The triangle ABC is called **equilateral** (equal sided) because it has two equal sides.

1. Draw a triangle : AB 3 in., BC 4 in. Measure AC.

2. $AB = 4$ in. $BC = 5$ in. $AC = ?$

3. $AB = 3$ miles. $BC = 6$ miles. $AC = ?$

Draw this by a scale 1 inch to 1 mile.

4. Draw another right angle triangle, AB 2 in., BC 8 inches. $AC = ?$ Upon each side of the triangle make a square divided into sq. in. Count all the sq. in.

QUESTIONS

1. How many times longer is a mile than a yard?
2. What is the ratio of a mile to a rod?
3. What is the ratio of a mile to 16 rods?
4. What is the ratio of a yard to an inch?
5. A boy's garden was 33 ft. wide by 66 ft. long. Give its dimensions in rods. Draw to scale $\frac{1}{4}$ in. = 1 ft.
6. One boy took a running jump and cleared 3 yd. Another jumped 100 inches. Which jumped farther? By how much?
7. A man walked 12 miles in 3 hr., and a second man walked 27 miles in $\frac{3}{8}$ of a day. Which one walked at the faster rate? What was the number of miles each walked in one hour?
8. What is the ratio of A's speed of 5 miles an hour and B's speed of 6 miles an hour in running? Be sure that your fraction which tells the ratio shows clearly which man runs the faster.

REVIEW

WRITE

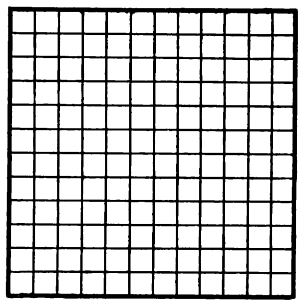
9. Review some of the additions on page 18.
10. Count by 24 to 200.
11. Write in words 251,863,007,521 and 70,001,989,502.
12. Subtract $5861.90 from $6000; $199,854 from $250,000; and $367.15 from $10,300.
13. To the scale of 1 in. to 1 ft. draw a square $5\frac{1}{2}$ ft. by 6 ft. Divide it by the scale into $\frac{1}{2}$ ft. squares.
14. Multiply 8432 by 2379 and prove by exchanging multiplier and multiplicand and multiplying again.
15. Divide 863,400 by 1000.
16. Prove by drawings that $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$.

SQUARE MEASURE

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ sq. ft.	= 1 square rod (sq. rd.)
160 square rods, or 43,560 sq. ft.	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

Square measure is used in measuring surfaces. A surface has only length and breadth; as, a sheet of paper, or one of the faces of a brick, or the face of a blackboard.

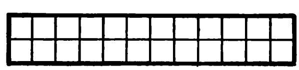
A square is a figure having four equal sides and four equal angles.



Scale $\frac{1}{8}$ in. to 1 in.

A square inch is a square each of whose sides is an inch long. A square foot is a square having each side 1 foot, or 12 inches long. Its contents are $12 \times 12 = 144$ square inches, that is, the square can be divided into 144 squares, each of whose sides is an inch long.

If the oblong or rectangle has sides 12 inches long and 2 inches wide, its square contents are equal to 12×2 inches, or 24 inches, that is, the rectangle is divisible into twenty-four squares, each of whose sides is an inch long. Hence, if the length and breadth of a square or of any other rectangle are given, the **area**, or contents, of the figure can be found by multiplying the length by the breadth.



Scale $\frac{1}{8}$ in. to 1 in.

Land, plastering, paving, painting, carpeting, and cloth are measured by square measure.

QUESTIONS

1. What is the area of a plot of ground 30 ft. by 180 ft.?
2. Of a plot 45 ft. by 120 ft.?
3. Of a plot 60 ft. by 100 ft.?
4. A man who wished to get for his children a plot of ground with as many square feet in it as possible, was offered three plots all at the same price. One was 90 ft. wide, 120 ft. deep; another was 50 ft. wide, 200 ft. deep; the third was 40 ft. wide, 300 ft. deep. Which was the best for him to buy?
5. Mr. Thompson bought a great farm containing 50 sq. m. How many acres was that?
6. A square 2 feet on a side contains 576 sq. in.

Show by a drawing to the scale of $\frac{12}{8}$ in. to 1 ft. why this is true. $\frac{12}{8}$ in. = how many inches?

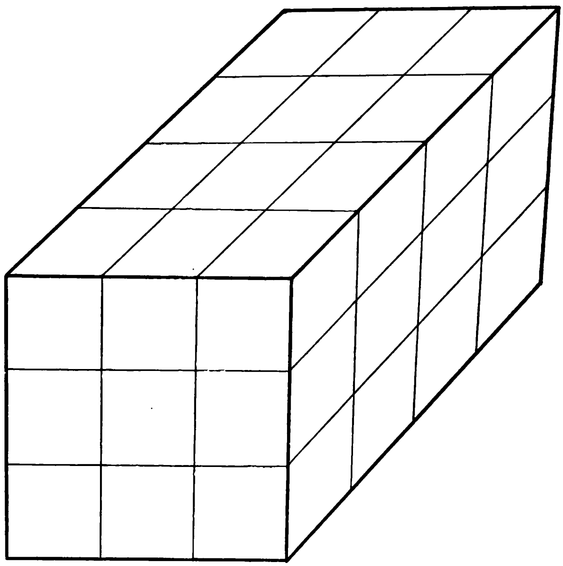
7. Which is the larger area, a 5 in. sq. or 20 sq. in.? By how much?
8. A carpet contained 28 sq. yd. If one side of the room was 12 ft. long, what was the length of the other side?
9. If a section of land contains 640 acres, how many acres does a quarter section contain? a half section?
10. What is the length of the perimeter of a square mile? of $\frac{1}{4}$ sq. in.?
11. A farmer sold 80 acres of land for $8000, but refused $10,000 for $\frac{1}{8}$ of a section. Which was the higher price? By what ratio was it higher?
12. A contractor offered to make a stone road bed at $30 a square rod. A competitor offered to make it at $1 a square yard. Which price was lower?
13. At the first price what would have been the cost of 1000 sq. rd.? What at the second price?
14. Write the aliquot parts of 100 divided in eighths.

CUBIC MEASURE

A **cube** has **length, breadth,** and **thickness**; six sides; and twelve edges, forming right angles with each other.

The sides are called **faces.**

The **volume** of a cube is found by multiplying together the numbers expressing its length, breadth, and thickness. The measures of the length, breadth, and thickness are called **dimensions.** The product is called **cubic inches, feet, yards,** etc.



Scale: ½ in. = 1 ft.

The dimensions of this cube are, if we follow the scale:

length	. . .	4 inches
breadth	. . .	3 inches
thickness	. .	3 inches
Its volume is	. . .	36 cubic inches

CUBIC MEASURE

Since a foot contains twelve inches, a cubic foot contains $12 \times 12 \times 12$ cubic inches. A cubic foot measures twelve inches in each dimension. $12 \times 12 \times 12 = 1728$.

Since a yard contains three feet, a cubic yard contains nine cubic feet.

TABLE

1728 cubic inches	=	1 cubic foot (cu. ft.)
27 cubic feet	=	1 cubic yard (cu. yd.)
128 cubic feet	=	1 cord (cd.), used in measuring wood

A **square prism** is a solid whose volume may be found by cubic measure. A **cord** of wood is piled in the form of a square prism 4 ft. by 4 ft. by 8 ft.

1. How many cubic inches are there in one cubic yard?

2. Find the number of cubic feet of space in a house 30 ft. wide, 40 ft. deep, and 22 ft. high.

3. A pile of wood measures 80 ft. by 40 ft. by 10 ft. How many cubic feet are there in its volume?

4. A fireplace holds 4 cu. ft. of wood. If this amount lasts an hour, how many hours does a cord of wood last?

5. In a coal mine a coal vein is 2 yd. thick, 5 yd. high, and 200 yd. long. How many cu. yd. are in the vein?

6. How many cu. in. are there in 100 cu. ft.?

7. How many cu. ft. of air are in a room 8 ft. $\times$ 8 ft. $\times$ 10 ft. in size?

8. How many cu. ft. are in a piece of lumber 24 ft. $\times$ 1 ft. $\times$ 9 in.?

CORD WOOD

Cord foot Cord

A cord of wood is as much wood as is contained in a pile measuring 4 ft. × 4 ft. × 8 ft.

A cord = 128 cu. ft. in space.

The wood is piled as it comes, and the space not actually taken by wood counts just as much as the solid wood.

A cord foot is 4 ft. × 4 ft. × 1 ft.

A cord foot = 16 cu. ft. of space.

1. How many cord feet are there in a cord?

2. Will's father bought 20 cords of wood. If this was piled 4 ft. wide and 8 ft. high, how long would the pile be?

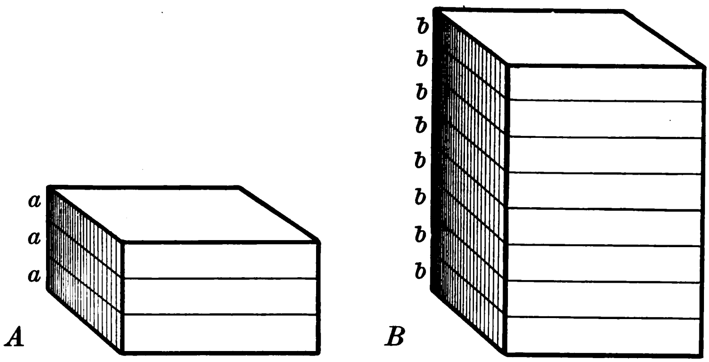
3. What part of a cord is 2 cord feet? 3 cord feet?

4. A pile of wood 4 ft. × 12 ft. × 12 ft. was offered to John Douglas at $5 a cord. He found the amount of the bill in this way:

$4 \times 12 \times 12 = 4 \times 3 \times 4 \times 3 \times 4 = 4 \times 4 \times 4 \times 3 \times 3 = \frac{1}{2}$ cord $\times 9 = \frac{9}{2}$ cords $= 4\frac{1}{2}$ cd. $\$5 \times 4\frac{1}{2} = \$20\frac{1}{2} = \$20.50$.

Can you follow these steps?

PRISMS



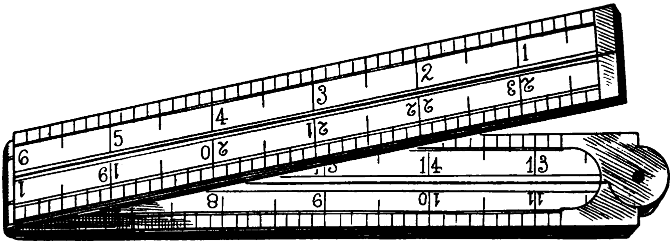
The *volume* or contents of a square *prism* are found by multiplying together the numbers which express its dimensions. A square prism is a cube all of whose faces are rectangles.

1. What is the ratio of *A* to *B*? of *B* to *A*?
2. If *A* is 100, what is *a*?
3. If *B* is 100, what is *b*?

The ratio of equals is one.

4. What is the ratio of *a* to *b*? of *b* to *a*?
5. If *a* is $33\frac{1}{3}\%$ of *A*, what is $a + a$?
6. If *b* is $12\frac{1}{2}\%$ of *B*, what is $b + b + b$?
7. Point out $87\frac{1}{2}\%$ of *B*.
8. Point out upon *B* 200% of *A*.
9. How would you draw $16\frac{2}{3}\%$ of *A*? What part of *A* would that be?
10. Draw prisms of various ratios to each other.

MEASURING



Folding Foot Rule

When we measure anything by long measure or square measure or cubic measure, we find its *ratio* to a fixed length, one inch, one foot, one yard, etc. Carpenters use "foot rules" and "2-foot rules."

When we say that a piazza on a house is twenty feet long, we mean that its ratio to one foot is twenty.

1. Draw a line on the blackboard which you think is 1 ft. long. Measure it.

2. Cut a strip of paper 1 ft. long by 1 in. wide. Mark on it inches divided into sixteenths.

3. Make estimates of windows, doors, floors, in feet and inches. Measure them, and compare the results.

4. Measure the width of sidewalks and streets, and bring the results to school.

5. Measure the sizes of buildings.

6. Make to scale outline drawings of floors, desks, and dictionaries.

7. Measure the length of the track of a wheel making one revolution.

REVIEW

1. A dealer sold 19 lambs for $76. How much apiece on an average did they cost the buyer?

2. A farmer secured 198 bushels of wheat from a seven-acre lot. How many bushels to an acre did the lot yield?

3. What is the ratio of 2 yd. to 3 in.?

4. How many times must we move and place a two-foot rule to measure two rods?

5. What is the ratio of a prism containing 32 cu. in. to another containing 640 cu. in.?

6. What is the ratio of a prism containing one cubic foot to another containing four hundred thirty-two cubic inches?

7. A farmer sold his farm of 137 acres for $8631. How much an acre did he get for the farm?

8. How many times is $17 contained in $11,917?

9. What sum of money taken 17 times will amount to $11,917?

10. Two dozen hens each laid in one year 12 dozen eggs. How many eggs in all were laid?

12 dozen make 1 gross.

11. A farmer cut in his woods one winter 125 cords. If the wood was piled 4 ft. wide and 4 ft. high, how long was the pile? How many cubic feet are there in 125 cords? This number divided by $4 \times 4 =$ what?

12. Which is greater, 5 cu. ft. or 1 cd.? By how much?

13. A grocer bought 3 gross of eggs at 10¢ a dozen. How much did he pay?

FRACTIONS

1. How many cents are there in $\frac{3}{4}$ of 100 cents? in $\frac{1}{4}$ of 300 cents? Is there any difference between $\frac{3}{4}$ of 1 and $\frac{1}{4}$ of 3?

2. What is $\frac{1}{4}$ of 8? $\frac{3}{4}$ of 8? $\frac{1}{4}$ of 24? $\frac{2}{3}$ of 6? $\frac{1}{3}$ of 12? $\frac{4}{5}$ of 10? $\frac{1}{5}$ of 40?

3. How many fourths are there in 1? in 2? in $2\frac{1}{4}$? in $3\frac{2}{4}$? in $6\frac{1}{4}$? in 12? in $12\frac{3}{4}$?

4. How many fifths are there in 1? in 3? in $3\frac{1}{5}$? in 5? in $5\frac{3}{5}$? in $10\frac{1}{5}$?

5. How many sixths are there in 1? in 2? in 4? in $5\frac{1}{6}$? in $9\frac{5}{6}$? in $11\frac{4}{6}$?

6. How many sevenths are there in 1? in 3? in 5? in $5\frac{1}{7}$? in $6\frac{2}{7}$? in $1\frac{1}{7}$?

7. How many eighths are there in 1? in 2? in $4\frac{1}{8}$? in $5\frac{2}{8}$? in $6\frac{1}{8}$?

8. Show by drawings that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

9. Show also that $\frac{3}{9} = \frac{1}{3}$ and that $\frac{3}{6} = \frac{1}{2}$.

REDUCTION OF FRACTIONS

Since $\frac{1}{2} = \frac{2}{4}$, and $\frac{1}{3} = \frac{2}{6}$, then multiplying both the numerator and the denominator of a fraction does not change its value, for while each fractional part is made smaller the number of parts taken is so increased that the ratio of the numerator to the denominator remains the same.

10. What is the ratio of 1 to 2? of 2 to 4?

11. What is the ratio of 1 to 3? of 3 to 6?

Multiplying both the numerator and the denominator of a fraction is one way of *reducing* a fraction, or changing its terms without changing its value.

EQUIVALENCE OF FRACTIONS

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1. $\frac{1}{2}=\frac{?}{4}=\frac{?}{8}=\frac{?}{16}=\frac{?}{32}$.

2. $\frac{1}{4}=\frac{?}{8}=\frac{?}{16}=\frac{?}{32}=\frac{?}{64}$.

3. $\frac{1}{8}=\frac{?}{16}=\frac{?}{32}=\frac{?}{64}=\frac{?}{128}$.

4. $\frac{1}{16}=\frac{?}{32}=\frac{?}{64}=\frac{?}{128}=\frac{?}{356}$.

5. $\frac{3}{4}=\frac{?}{8}=\frac{?}{16}=\frac{?}{32}$.

6. $\frac{3}{8}=\frac{?}{16}=\frac{?}{32}=\frac{?}{64}$.

7. $\frac{5}{8}=\frac{?}{16}=\frac{?}{32}=\frac{?}{64}$.

8. $\frac{7}{8}=\frac{?}{16}=\frac{?}{32}=\frac{?}{64}$.

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Show by drawings that

9. $\frac{1}{3}=\frac{3}{9}=\frac{2}{6}=\frac{5}{15}$.

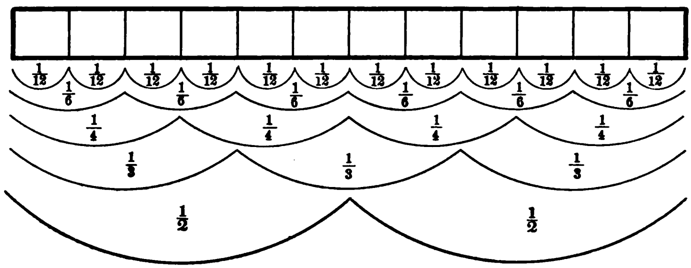
10. $\frac{1}{3}=\frac{?}{27}=\frac{?}{18}=\frac{?}{30}$.

11. $\frac{2}{9}=\frac{?}{27}$.

12. $\frac{2}{15}=\frac{?}{30}$.

13. Write in order of size: $\frac{1}{27}$, $\frac{1}{18}$, and $\frac{1}{30}$.

TWELFTHS



$\frac{2}{12} = \frac{1}{6}$ $\frac{3}{12} = \frac{1}{4}$ $\frac{4}{12} = \frac{1}{3}$ $\frac{6}{12} = \frac{1}{2}$

1. If anything is divided into any number of parts, what ratio of the numerator to the denominator always indicates the whole thing?

2. If we change the numerator of a fraction by multiplying it, why must we also always multiply the denominator by the same number?

3. In these fractions, what is the ratio of the denominators to the numerators: $\frac{3}{12}$, $\frac{1}{4}$, $\frac{6}{24}$?

4. How many eighths are there in one fourth? how many twelfths? how many sixteenths?

5. $\frac{4}{6} = \frac{?}{12}$ $\frac{3}{12} = \frac{?}{4}$ $\frac{1}{3} = \frac{?}{12}$ $\frac{9}{12} = \frac{?}{4}$

6. Draw a figure representing $\frac{12}{12}$. If each twelfth should represent a sixth of the figure, how many wholes would this figure represent?

7. If each twelfth should represent a fourth, how many wholes would we have?

8. $\frac{12}{3} = ?$ $\frac{12}{2} = ?$ $\frac{12}{8} = ?$ $\frac{12}{9} = ?$

FRACTIONS

Since $\frac{2}{4} = \frac{1}{2}$ and $\frac{2}{6} = \frac{1}{3}$, then dividing both the numerator and the denominator of a fraction does not alter its value, for while each fractional part is made larger, the number of parts taken is so decreased that the ratio of the numerator to the denominator is unchanged.

Dividing both the numerator and the denominator of a fraction is another way of *reducing* a fraction, or changing its terms without changing its value.

1. Reduce $\frac{1}{4}$ to eighths, $\frac{1}{3}$ to ninths, $\frac{1}{5}$ to tenths.

2. Reduce $\frac{1}{6}$ to twelfths, $\frac{1}{8}$ to sixteenths, $\frac{1}{5}$ to twentieths.

3. Reduce $\frac{1}{10}$ to hundredths, $\frac{1}{20}$ to hundredths, $\frac{1}{5}$ to hundredths.

4. Reduce to fifths these fractions: $\frac{4}{10}$, $\frac{15}{25}$, $\frac{60}{100}$.

5. Reduce to eighths these fractions: $\frac{6}{24}$, $\frac{16}{48}$, $\frac{8}{32}$.

6. Reduce to hundredths these fractions:

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{25}$, $\frac{1}{50}$, $\frac{3}{4}$, $\frac{3}{25}$, $\frac{8}{25}$, $\frac{12}{25}$, $\frac{21}{25}$.

REDUCTION OF FRACTIONS

Multiplying or dividing the terms of a fraction by the same number does not change the value of a fraction.

$\$\frac{2}{4} = ?$ Dividing both terms by 2, we obtain $\$\frac{1}{2}$. $\frac{2}{4}$ of a dollar = 50¢. $\frac{1}{2}$ of a dollar = 50¢. $\$\frac{2}{4} = \$\frac{1}{2}$.

$\frac{6}{9} = \frac{2}{3}$. We divide both numerator and denominator of $\frac{6}{9}$ by 3.

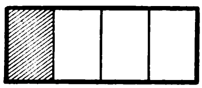
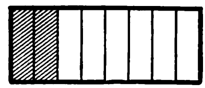
$\frac{6}{9}$ of 18 oranges $= 6 \times \frac{1}{9}$ of 18 oranges. $\frac{1}{9}$ of 18 oranges = 2 oranges. $\frac{6}{9}$ of 18 oranges $= 6 \times 2$ oranges = 12 oranges.

$\frac{1}{3}$ of 18 oranges = 6 oranges. $\frac{2}{3}$ of 18 oranges = 2×6 oranges = 12 oranges.

REDUCING FRACTIONS

Sometimes it is convenient for us in working with a fraction to have its denominator larger. This means that the unit is to be divided into more equal parts.

The real value of the fraction remains the same.

 $\frac{1}{4} = \frac{2}{8}$ 

To reduce fractions to higher terms, we multiply both the denominator and the numerator by a number larger than one. Higher terms mean more equal parts.

1. By what number is $\frac{1}{4}$ multiplied in reducing it to $\frac{2}{8}$?

2. Reduce $\frac{3}{5}$ to 15ths. $15 = 5 \times 3$. $\frac{3}{5} = \frac{9}{15}$.

To make the denominator 15, we multiply it by 3. If we multiply the numerator also by 3, we do not change the value of the fraction.

3. Reduce $\frac{7}{17}$ to 68ths.

$$17\overline{)68}(4 \quad 68 \qquad \frac{7 \times 4}{17 \times 4} = \frac{28}{68} \qquad \text{Proof: } \frac{4)28}{4)68} = \frac{7}{17}$$

4. Reduce $\frac{11}{15}$ to a fraction whose denominator is 90.

5. Reduce $\frac{10}{27}$ to a fraction whose denominator is 54.

6. Change 9 to a fraction whose denominator is 3.

$$1 = \frac{3}{3} \quad 9 = 1 \times 9 \quad \frac{3}{3} \times 9 = \frac{27}{3} \quad 9 = \frac{27}{3}$$

7. Change 9 to a fraction whose denominator is 11.

Reduce:

8. $\frac{5}{8}$ to 72ds. **9.** $\frac{3}{7}$ to 49ths. **10.** $\frac{4}{9}$ to 27ths.

11. $\frac{2}{9}$ to 81sts. **12.** $\frac{12}{15}$ to 150ths. **13.** $\frac{3}{10}$ to hundredths.

14. $\frac{4}{7}$ to 63ds. **15.** $\frac{16}{64}$ to 128ths. **16.** $\frac{5}{100}$ to thousandths.

REVIEW

1. A boy collected 125 foreign postage stamps at an average cost of 4¢ each. How much money did he spend?

2. 5 of the stamps cost $1.40. How much in all did the other stamps cost?

3. What was the average cost of the 5 stamps? What was the average cost also of the 120 other stamps?

4. Count by 16 to 200.

5. Read:

$782 039 648 521; 84 326 800 000; 100 600 000 000.

6. The wealth of the United States in 1900 was about ninety billion dollars. How many figures including zeros must we write to express this?

7. If the average wealth of all the persons in the United States in 1900 was $1200, what was the average wealth of the average family of five persons?

8. How do we multiply by 10, 100, 1000 most easily?

9. What fractional part of a dollar is 5¢? 10¢? 25¢?

10. What is 14×12? 15×11? 17×12? 16×11?

11. What is the volume of a prism 8 ft. × 5 ft. × 5 ft.?

12. What is the volume of a prism $12\frac{1}{2}$ ft. × 4 ft. × 2 ft.?

13. What is the ratio of the volume of the prism in **12** above to that of the prism in **11**?

14. What fraction with a smaller denominator is equivalent to $\frac{3}{24}$? $\frac{2}{16}$? $\frac{4}{20}$?

15. Give the tables for Long Measure, Square Measure, and Cubic Measure.

16. Tell the meanings of: **area, scale, addend, minuend, subtrahend, multiplicand, dividend, quotient, fraction, ratio, per cent, prism, cubic measure, cord.**

REVIEW

1. If a clock ticks 3600 times an hour, how many times will it tick in two days of 24 hours each?

2. Divide a tax of $53,400 equally among 12 towns.

3. The quotient is 40, the divisor is 364, and the remainder is 120. What is the dividend?

4. A man's salary is $780 per annum. What is it per month? per week?

5. The sum of $4824 is to be raised from 12 counties, in each of which are 6 towns. How much must be raised in each town?

6. Twenty-one casks contain 10,941 herrings. How many herrings are there on the average in each cask?

7. The earth moves around the sun at the rate of 66,600 miles an hour. How many miles is that per minute?

8. Reduce $\frac{4}{7}$ths to 49ths, and add $\frac{6}{49}$ths. What is the result?

9. In round numbers what is the earth's circumference? How many days would it take to travel around the earth, going at the rate of 125 miles a day?

10. Reduce to smaller denominations: $\frac{8}{14}$, $\frac{6}{18}$, $\frac{10}{28}$, $\frac{12}{48}$.

11. There are eight houses in a village, and each house has seventeen windows. How many windows are there in all the houses? At 6 panes to a window, how many are there in all?

12. A man who had 6 cords of wood made a pile 8 ft. high and 12 ft. wide. How long was it?

13. A public park contained 919,960 sq. ft. It was 155 ft. wide. How long was it?

14. A boy ran 1000 yd. Was this more than $\frac{1}{2}$ mi.?

IMPROPER FRACTIONS

We know that any fraction in which the numerator is larger than the denominator is made up of two numbers, a whole number and a fraction, because an improper fraction takes more parts than a single unit is divided into. $\frac{8}{4}$ means 2 units, since 1 unit contains 4 fourths. $8 = 4 \times 2$.

A whole number is called an **integer**.

1. How many units are there in $\frac{16}{4}$?

Since in 4 fourths there is 1 unit, in 16 fourths there are as many units as there are 4 fourths in 16 fourths: that is, 4 units. Hence $\frac{16}{4} = 4$.

2. How many units are there in $\frac{22}{3}$?

$\frac{22}{3}$ = as many units as there are $\frac{3}{3}$ in $\frac{22}{3}$, or $7\frac{1}{3}$ units.

3. Reduce $\frac{89}{12}$ to a mixed number.

$$\frac{89}{12} = 89 \div 12 = 7\frac{5}{12}$$

4. Change $\frac{197}{37}$ to a mixed number.

$$\begin{array}{r} 37)197(5\frac{12}{37} \\ \underline{185} \\ 12 \end{array}$$

5. Reduce these fractions to whole or mixed numbers:

a. $\frac{15}{4} = ?$ *b.* $\frac{58}{8} = ?$ *c.* $\frac{41}{9} = ?$ *d.* $\frac{442}{7} = ?$ *e.* $\frac{108}{7} = ?$

f. $\frac{19}{8} = ?$ *g.* $\frac{75}{10} = ?$ *h.* $\frac{311}{12} = ?$ *i.* $\frac{15}{2} = ?$ *j.* $\frac{26}{5} = ?$

6. Reduce to dollars and cents these numbers:

a. $\$\frac{20}{3}$. *b.* $\$\frac{67}{5}$. *c.* $\$\frac{100}{12}$. *d.* $\$\frac{50}{6}$. *e.* $\$\frac{35}{8}$.

f. $\$\frac{9}{4}$. *g.* $\$\frac{28}{3}$. *h.* $\$\frac{25}{6}$. *i.* $\$\frac{49}{8}$. *j.* $\$\frac{14}{3}$.

IMPROPER FRACTIONS

The reason why fractions larger than a whole unit are called improper fractions is this: a fraction is an equal part of anything, and no part can ever be larger than the whole. It is not *proper*, because of these definitions, to call anything larger than a whole a fraction at all. But it is often convenient to do so, and we call numbers that are partly fractional *improper* fractions.

1. Reduce $3\frac{4}{5}$ to 5ths.

$$3\tfrac{4}{5} = \tfrac{15}{5} + \tfrac{4}{5} = \tfrac{19}{5}$$

Since there are 5 fifths in 1, in 3 there are 3 times 5 fifths, or 15 fifths; and 15 fifths + 4 fifths = $\frac{19}{5}$.

2. Reduce $15\frac{6}{7}$ to 7ths.

$$\tfrac{15}{1} \times \tfrac{7}{7} = \tfrac{105}{7} \qquad \tfrac{105}{7} + \tfrac{6}{7} = \tfrac{111}{7}$$

3. Reduce $20\frac{2}{13}$ to 13ths.

$$\tfrac{20}{1} \times \tfrac{13}{13} = \tfrac{260}{13} \qquad \tfrac{260}{13} + \tfrac{2}{13} = \tfrac{262}{13}$$

4. How many 3ds are there in $3\frac{1}{3}$? $1\frac{2}{3}$? $2\frac{1}{3}$? $3\frac{2}{3}$?

5. How many 5ths are there in 1? in $1\frac{1}{5}$? in 2? in $1\frac{2}{5}$? in $2\frac{2}{5}$? in $4\frac{1}{5}$?

6. How many 7ths are there in 2? in 3? in $2\frac{3}{7}$? in $3\frac{6}{7}$? in $3\frac{1}{7}$? in $5\frac{2}{7}$?

7. How many quarter dollars are there in \$2? \$$5\frac{1}{4}$? \$$3\frac{3}{4}$? \$$6\frac{1}{4}$? \$10? \$75?

8. Change to improper fractions: $3\frac{1}{5}$; $2\frac{1}{5}$; $6\frac{1}{6}$; $5\frac{1}{11}$; $4\frac{7}{12}$; $7\frac{2}{5}$; \$$10\frac{1}{2}$; \$$25\frac{5}{8}$.

9. Change to 9ths: $3\frac{1}{9}$; $2\frac{4}{9}$; $7\frac{7}{9}$; $8\frac{4}{9}$; $9\frac{1}{9}$; $11\frac{1}{9}$.

10. How many 10ths are there in 1? $1\frac{3}{10}$? $1\frac{6}{10}$? 2? $5\frac{4}{10}$? 100? 200? 500?

1. Twelve twelfths. $1 = \frac{12}{12}$.

2. One one half

$1\frac{1}{2} = \frac{3}{2}$.

3. Twelve eighths. $\frac{12}{8} = \frac{3}{2}$.

4. One one third

$1\frac{1}{3} = \frac{4}{3}$.

5. Four thirds. $\frac{4}{3}$.

6. Twelve ninths. $\frac{12}{9} = \frac{4}{3}$.

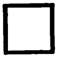
7. Six sixths. $\frac{6}{6} = 1$.
Six fifths. $\frac{6}{5} = 1\frac{1}{5}$.
Six fourths. $\frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$.
Six thirds. $\frac{6}{3} = 2$.
Six halves. $\frac{6}{2} = 3$.
Six. $6 = 6$ ones.

8. Show that $\frac{12}{10} = 1\frac{2}{10} = 1\frac{1}{5}$.

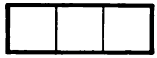
A mixed number is a unit, or several or many units, with a fraction of the unit added.

9. Draw an inch and seven eighths; and $\frac{35}{16}$ inches.

RATIOS AND FRACTIONS


A


B


C

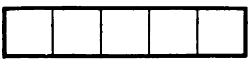

D

1. What is the ratio of A to B? of B to A? of A to C? of B to C?

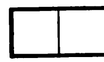
2. Why is the ratio of B to C $\frac{2}{3}$ and of C to B $\frac{3}{2}$?

3. What is the ratio of A to D? of D to A? of B to D? of D to B?

4. Why is the ratio of C to D $\frac{3}{4}$ and of D to C $\frac{4}{3}$?

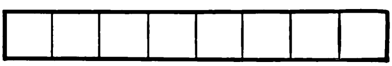

E


F

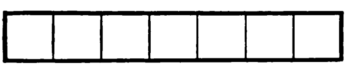

B

5. What is the ratio of B to E? of E to B? of B to F? of F to B?

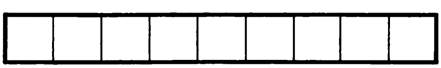
6. What fraction of E is C? is D? of F is C? is D?


H

7. What is the ratio of G to H? of H to G? of D to H? of H to D?


G

8. What fraction of H is B? is E? is F?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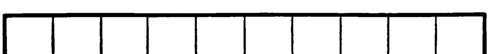

J

9. What forms show the ratio $\frac{7}{10}$? $\frac{5}{9}$? $\frac{3}{8}$? 4? $\frac{10}{6}$? $\frac{1}{2}$? $\frac{1}{3}$?

10. Fold paper to show fractions and ratios like these.

11. Make drawings of larger sizes, showing similar ratios and fractions.

RATIO AND FRACTION DRAWINGS

Notice the fraction drawings on pages 71, 72. They are made with exactness. But in showing the principles of fractions, outside of books, it is not necessary to make drawings with equal exactness. Freehand drawings, made without rulers or straight edges, show the truth sufficiently well.

Circles may be made by tracing around nickel or silver coins or spools. Other methods are explained on page 118.

1. Show the equivalence of $\frac{3}{8}$, $\frac{6}{16}$, and $\frac{9}{24}$ by drawings.
2. Show the ratios of 3, 5, 8, 10 to each other by drawings.
3. Show by drawings the ratios $\frac{3}{5}$, $\frac{3}{8}$, $\frac{3}{10}$.
4. Show also the ratios $\frac{5}{3}$, $\frac{5}{8}$, $\frac{5}{10}$.
5. Show the equivalence of $\frac{4}{5}$ and $\frac{8}{10}$.
6. Show by a drawing the ratio of a gallon to a quart.
7. Show also the ratio of a peck to a quart.
8. Show the ratio of a foot to a yard.
9. Show the ratio of $2\frac{1}{2}$ to $\frac{1}{4}$.
10. Show the ratio of $\frac{9}{8}$ to 1.

GENERAL REVIEW — WRITE

11. Add : $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{10}$; $\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{9}$, and $\frac{1}{12}$.
12. Subtract : $\frac{7}{12}$ from $\frac{11}{15}$; $\frac{5}{6}$ from $\frac{24}{25}$.
13. Add : $\$2\frac{1}{3}$, $\$5\frac{2}{5}$, and $\$8\frac{3}{4}$.
14. Reduce to mixed numbers : $\frac{30}{9}$, $\frac{8}{3}$, $\frac{43}{7}$, and $\frac{29}{10}$.
15. Reduce to improper fractions : $2\frac{1}{6}$, $8\frac{3}{8}$, and $1\frac{83}{100}$.

FACTORS

The **factors** of a number are the numbers that multiplied together produce the number.

The factors of 12 are 4 and 3, 6 and 2, or 2, 2, and 3; for $4 \times 3 = 12$, $6 \times 2 = 12$, or $2 \times 2 \times 3 = 12$.

As the factors, 2, 2, 3, 4, and 6, will exactly divide 12, those numbers are also **divisors** of 12.

An **integer**, or an integral number, is a whole number. 1, 6, 8, 22, are integers.

$6\frac{1}{2}$, $3\frac{7}{8}$, $\frac{3}{4}$, $\frac{7}{11}$, $12\frac{1}{9}$, are not integers, but **mixed numbers**, since they are made up of integers and fractions.

A **prime number** is a number which has no integral factor except itself and 1.

3, 5, 7, 11, 13, 19, are prime numbers.

Numbers are prime to one another when no number but 1 will exactly divide each of them. Thus, 3 and 5; 4 and 7; 8 and 19; 21 and 2; 6, 11, 17, and 25, are prime to each other.

A **prime factor** is a factor that is a prime number.

2, 2, and 3 are the prime factors of 12. The prime factors of 18 are $3 \times 2 \times 3$. The prime factors of 24 are $2 \times 2 \times 2 \times 3$.

A number that is a factor of two or more numbers is called a **common factor**, or a common divisor of the numbers.

2 is a common factor, or a common divisor, of 6, 12, 14, 18, 20, 22, because it will exactly divide each of them.

A **composite number** is a number that has integral factors besides itself and 1.

$4 = 2 \times 2$, $9 = 3 \times 3$, $14 = 2 \times 7$, $39 = 3 \times 13$, $55 = 5 \times 11$, $100 = 2 \times 2 \times 5 \times 5$, $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$.

FACTORS

1. Name the eight factors of 24.

2. Give all the factors of 4, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30.

3. What is a factor? a prime number? a composite number? Give two numbers of each kind.

4. What number is always a factor of even numbers?

5. Is there any prime number above 2 which is not an odd number?

6. Find all the prime numbers from 3 to 53.

7. Name all the factors of 33, 35, 39, 42, 45, 50.

8. Name all the composite numbers from 48 to 60.

9. What prime factor is common to

6 and 9?	10 and 12?	15 and 20?	11 and 22?
16 and 24?	24 and 8?	30 and 35?	28 and 14?

10. What is the largest factor common to

12 and 24?	10 and 30?	20 and 36?	24 and 36?
36 and 27?	16 and 32?	26 and 13?	40 and 60?

11. What is a factor? A prime factor? A common factor? A composite number? Give an example of each.

12. What factor is common to the denominators of these fractions: $\frac{2}{3}$, $\frac{4}{21}$, $\frac{8}{9}$, $\frac{1}{24}$?

13. What factor is common to the numerators of these fractions: $\frac{12}{15}$, $\frac{8}{9}$, $\frac{24}{25}$, $\frac{60}{100}$?

14. $2 \times 3 \times 5 = ?$ $2 \times 2 \times 3 \times 3 = ?$ $3 \times 5 \times 7 = ?$ Why are these composite numbers?

TESTS FOR FACTORS

Any number is divisible by 2 if the unit figure is divisible by 2. Then 12, 16, 18, 152, 374, are divisible by 2, because 6, 8, 2, and 4 can be divided by 2.

Any number is divisible by 3 if the sum of its **digits,** or figures, is divisible by 3.

474 can be divided by 3, for its digits, added together, make a sum, 15, which is divisible by 3. $(4+7+4=15.)$

393 can be divided by 3, for $3+9+3=15$, a number divisible by 3.

1866 is divisible by 3, for $1+8+6+6=21$, a number of which 3 is a factor.

A number is divisible by 4 if the first two right-hand figures (units and tens) are divisible by 4.

144, 288, 824, can each be divided by 4, for 44, 88, and 24 are divisible by 4.

A number is divisible by 5 if its final digit is 5 or 0.

A number is divisible by 9 if the sum of its digits is divisible by 9.

738 is divisible by 9, for $7+3+8=18$, a number of which 9 is a factor. 117 is divisible by 9, for $1+1+7=9$, and $9=9\times 1$.

1. Find the prime factors of 825.

$$\begin{array}{r|r} 5 & 825 \\ \hline 5 & 165 \\ \hline 3 & 33 \\ \hline & 11 \end{array}$$

$$825=5\times 5\times 3\times 11$$

2. What are the prime factors of 180?

$$\begin{array}{r|r} 5 & 180 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$5\times 2\times 2\times 3\times 3=180$$

GREATEST COMMON DIVISOR

A **common divisor** of two or more numbers is a number that will exactly divide each of them.

3 is a common divisor or factor of 9, 27, and 81. 3 divides each of these numbers without remainder.

The **greatest common divisor** of two or more numbers is the greatest number that exactly divides each of them.

9 is the greatest common divisor of 9, 27, and 81. 9 is the greatest number that exactly divides each of these numbers.

Numbers that have no common divisor, except 1, are said to be **prime** to one another.

14 and 15 are prime to each other. So also are 9 and 16, 8 and 21, 12 and 25.

$$\begin{array}{r|rrr} 3 & 12 & 24 & 36 \\ \hline 2 & 4 & 8 & 12 \\ \hline 2 & 2 & 4 & 6 \\ \hline & 1 & 2 & 3 \end{array}$$

1. What is the greatest common divisor of 12, 24, and 36?

$$3 \times 2 \times 2 = 12$$

3 is a common factor of 12, 24, and 36.

2 is a common factor of the quotients 4, 8, and 12.

Again, 2 is a common factor of the quotients 2, 4, and 6. But the quotients 1, 2, and 3 are prime to one another. Hence the product of the three common divisors, $3 \times 2 \times 2 = 12$, is the greatest common divisor.

$$\begin{array}{r|rrr} 3 & 42 & 63 & 105 \\ \hline 7 & 14 & 21 & 35 \\ \hline & 2 & 3 & 5 \end{array}$$

2. What is the greatest common divisor of 42, 63, 105?

$$3 \times 7 = 21$$

$$\begin{array}{r|rrrr} 2 & 24 & 40 & 64 & 72 \\ \hline 2 & 12 & 20 & 32 & 36 \\ \hline 2 & 6 & 10 & 16 & 18 \\ \hline & 3 & 5 & 8 & 9 \end{array}$$

3. What is the greatest common divisor of 24, 40, 64, 72?

$$2 \times 2 \times 2 = 8$$

GREATEST COMMON DIVISOR

What is the greatest common divisor of

1.	6 and 10?	**2.**	24 and 18?	**3.**	51 and 34?
4.	12 and 20?	**5.**	18 and 27?	**6.**	24 and 32?
7.	12 and 16?	**8.**	24 and 36?	**9.**	132 and 121?
10.	15 and 25?	**11.**	18 and 48?	**12.**	15, 7, and 105?
13.	15 and 18?	**14.**	42 and 28?	**15.**	60, 96, 108?
16.	10, 30, 40, 60?			**17.**	8, 12, 36?

18. What is the greatest common factor, or divisor, of 12, 24, 36, 48? of 18, 45, 54, 63?

19. What is the largest number of boys among whom 437 apples and 1691 peaches can be divided equally?

20. The principal of a school, with classes of boys numbering 45, 54, and 36, wished to divide them up into equal squads for drill. What was the largest number he could place in squads of exactly equal size in each class, keeping the different classes separate?

21. Nine boys had marbles: the first, 21; the second, 35; the others 63, 84, 49, 14, 77, 28, and 42. They divided them into sets of equal number. Each boy had only his own marbles in his sets. How large was each set? How many sets had each of these nine boys?

22. What is the largest number that evenly divides 126, 189, 225, and 297?

The usefulness of the greatest common divisor is very limited. We seldom need to know the greatest common divisor of several numbers. But the operation to find it is ingenious and attractive and yet easy. It is desirable to have some attention given to it, if only as a mental diversion or mathematical amusement.

LEAST COMMON MULTIPLE

A **multiple** of a number is a number that is exactly divisible by it. 12 is a multiple of 3.

A **common multiple** of two or more numbers is a number that can be exactly divided by each of them. 32 is a multiple of 8 and 16.

The **least common multiple** of two or more numbers is the least number that can be exactly divided by each. 24 is the least common multiple of 12, 6, and 8.

1. Let us find the least common multiple of 4, 6, 8, and 12.

$$12 = 3 \times 2 \times 2 \qquad 6 = 3 \times 2$$
$$8 = 2 \times 2 \times 2 \qquad 4 = 2 \times 2$$

The least common multiple must contain all the factors of the largest number, 12, or $3 \times 2 \times 2$.

The least common multiple must contain 8, or its factors, $2 \times 2 \times 2$.

For the factors of 12 we have two 2's and 3. $12 = 2 \times 2 \times 3$.

The least common multiple must also contain the factors 6 and 4. The factors of these numbers we already have in the 12. The least common multiple, therefore, is $3 \times 2 \times 2 \times 2 = 24$.

Proof: $24 \div 12 = 2$. $24 \div 8 = 3$. $24 \div 6 = 4$. $24 \div 4 = 6$.

2. Find the least common multiple of 10, 15, 16, 24, 32.

$$32 = 2 \times 2 \times 2 \times 2 \times 2 \qquad 15 = 3 \times 5$$
$$24 = 2 \times 2 \times 2 \times 3 \qquad 10 = 2 \times 5$$
$$16 = 2 \times 2 \times 2 \times 2$$

As 32 contains all the factors of all the other numbers, except the factors 3 and 5, the least common multiple must be $32 \times 3 \times 5 = 480$.

LEAST COMMON MULTIPLE

Proof that 480 contains 10, 15, 16, 24, and 32 each without a remainder.

```
32)480(15      24)480(20      16)480(30      15)480(32
   32             48             48             45
   ---            --             --             --
   160                                           30
   160          480 ÷ 10 = 48                    30
   ---                                           --
```

Find the least common multiples of:

a. 14, 28, 21.

b. 16, 24, 48.

c. 3, 6, 9, 18.

d. 5, 15, 30, 40.

e. 5, 10, 90, 45.

f. 3, 9, 24, 36, 54.

g. 6, 9, 18, 27, 54.

h. 5, 10, 25, 30, 45, 75, 90.

i. 15, 18, 20, 45, 54, 60.

j. 2, 4, 6, 8, 9, 12, 16.

k. 12, 16, 20, 30, 36, 60, 72.

l. 4, 12, 36, 48, 60, 120.

REVIEWS

1. Find the G.C.D. of 320, 24, 72, 60, and 120.
2. At $1 per M. cu. ft. of gas, what was the December bill for a family that used $2\frac{3}{4}$ M. cu. ft.?
3. Find the prime factors of 1200, 720, 3000, 999.
4. What per cent is 12 of 72? 60? 96? 120?
5. What is the ratio of 6 bu. to 2 qt.? 1 pk.? 1 pt.?
6. Divide 7258 by 48; 1,252,825 by 125.
7. Multiply 10,500 by 678; 569; 431; 287.
8. Add $\$3\frac{3}{4}$, $\$2\frac{1}{2}$, $\$4\frac{1}{2}$, $\$5\frac{1}{4}$.
9. What is the ratio of 16 cu. ft. of air to 480 cu. ft.?
10. Subtract 15,879 from 104,798.
11. Draw a right angle triangle. Measure its sides.

REVIEW

1. How many pecks are there in 8 qt.? 32 qt.? 24 qt.?

2. What is the ratio of 10 gal. to 32 qt.?

3. A boy with 50¢ desired to buy oranges at 20¢ a dozen. How many dozen could he get?

4. What part of a number is 100% of it?

5. A man bought a house for $5000 and sold it at a gain of 10%. How much money did he gain?

6. An orchard containing 10,656 trees consisted of 96 rows. How many trees were there in each row?

7. In twelve weeks a printer had set 3,993,276 letters. How many letters a day did he set?

8. What are the prime factors of 33, 68, 92?

9. A yard of ribbon sells for $$\frac{1}{2}$. What is the price of $\frac{1}{2}$ a yard?

10. If a pound of tea cost 60¢, how much will a quarter of a pound cost?

11. If a man earns $90 a month and saves $\frac{1}{10}$ of it, in how many months will he save $63?

12. A grove contained 398 trees. Fire destroyed 226 of them. How many trees were left?

13. One man walked forty miles, a second man walked sixty-one miles, and a third man walked thirty-eight miles. How many miles altogether did the three men walk?

14. Two men start from the same point and travel in exactly opposite directions; after one man has journeyed five hundred and sixty-eight miles, and the other man three hundred and eleven miles, how many miles are the two men apart?

G. C. D. AND L. C. M.

The same process will give both the greatest common divisor and the least common multiple.

1. Find the greatest common divisor of 36, 60, 96, and 144.

$$\begin{array}{r|rrrr} 2 & 36 & 60 & 96 & 144 \\ \hline 2 & 18 & 30 & 48 & 72 \\ \hline 3 & 9 & 15 & 24 & 36 \\ \hline & 3 & 5 & 8 & 12 \end{array}$$

The common divisors are 2, 2, 3.

$$2 \times 2 \times 3 = 12$$

The greatest common divisor is 12.

2. Find the least common multiple of 36, 60, 96, and 144.

The divisors common to two or more of these numbers are 2, 2, 3, 2, 3, 2. The prime dividends are 5, 2.

$$\begin{array}{r|rrrr} 2 & 36 & 60 & 96 & 144 \\ \hline 2 & 18 & 30 & 48 & 72 \\ \hline 3 & 9 & 15 & 24 & 36 \\ \hline 2 & 3 & 5 & 8 & 12 \\ \hline 3 & 3 & 5 & 4 & 6 \\ \hline 2 & 1 & 5 & 4 & 2 \\ \hline & 1 & 5 & 2 & 1 \end{array}$$

$$\begin{array}{l} 2 \times 2 \times 3 \times 2 \times 3 \times 2 \times 5 \times 2 = 1440 \\ \quad 4 \quad 12 \quad 24 \quad 72 \quad 144 \quad 720 \quad 1440 \end{array}$$

To find the least common multiple we extend the process of finding the common divisors and multiply all the prime factors together.

Proof of 1

$$\begin{array}{r|rrrr} 12) & 36 & 60 & 96 & 144 \\ \hline & 3 & 5 & 8 & 12 \end{array}$$

No larger number than 12 will divide all the numbers, since the dividends of 36 and 60 by 12 as divisor are prime to each other.

Proof of 2

$$\begin{array}{llll} 36)1440(40 & 60)1440(24 & 96)1440(15 & 144)1440(10 \\ \quad\underline{144} & \quad\underline{120} & \quad\underline{96} & \\ \qquad\ \ 0 & \qquad 240 & \quad 480 & \\ \qquad\ \ \underline{0} & \qquad \underline{240} & \quad \underline{480} & \end{array}$$

No smaller number than 1440 will contain all these numbers, since 40, 24, 15, and 10 have no common factor.

REVIEW

1. What is the least common multiple of 4, 6, 8?

2. What is the greatest common divisor of 4, 6, 8?

3. What are the prime factors of 6, 8, 9, 10, 12?

4. If the units' figure of a number may be divided by 2, what do we know is one factor of the number?

5. If the sum of the digits of a number may be divided by 3, what do we know of the factors of a number?

6. If the final digit is 0 or 5, what do we know of the number?

7. What is a prime factor?

8. Give some numbers that have to each other the ratios of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$.

9. Reduce to improper fractions: $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$, $5\frac{2}{5}$, $6\frac{5}{6}$.

10. Reduce to fifths these fractions: $\frac{20}{30}$, $\frac{40}{50}$, $\frac{80}{100}$.

11. What is the volume of a prism $5 \times 5 \times 8$?

12. What is the volume of a prism of which the ratio to the above prism is 5?

13. If the ratio were $\frac{1}{5}$ of the first prism, what would the volume of a third prism be?

14. The size of a garden is 15 ft. $\times$ 20 ft. How many feet of wire netting must the gardener get to protect the garden from fowls and animals?

15. What is the ratio of one million dollars to one billion? of a billion to one million?

16. What per cent is $\frac{1}{6}$ of anything? $\frac{1}{8}$ of it?

17. Which is greater, $\frac{2}{3}$ or 70% of anything? By how much?

ADDITION OF FRACTIONS

Add:

1. $\frac{3}{5}+\frac{7}{5}+\frac{2}{5}$. **2.** $\frac{4}{11}+\frac{1}{11}+\frac{7}{11}$. **3.** $\frac{1}{3}+\frac{5}{3}+\frac{7}{3}$.

4. $\frac{3}{10}+\frac{7}{10}+\frac{8}{16}+\frac{11}{10}$. **5.** $\frac{6}{101}+\frac{20}{101}+\frac{39}{101}$. **6.** $\frac{5}{9}+\frac{10}{9}+\frac{11}{9}$.

7. Add: $\frac{5}{6}$, $\frac{3}{8}$, $\frac{5}{12}$, and $\frac{1}{4}$.

$$12 = 3 \times 2 \times 2$$
$$8 = 2 \times 2 \times 2$$
$$6 = 3 \times 2$$
$$4 = 2 \times 2$$

or

$$\begin{array}{r|rrrr} 2 & 4 & 6 & 8 & 12 \\ \hline 2 & 2 & 3 & 4 & 6 \\ \hline 3 & 1 & 3 & 2 & 3 \\ \hline & 1 & 1 & 2 & 1 \end{array}$$

The least common multiple of the denominators is:

$$3 \times 2 \times 2 \times 2 = 24 \text{ or } 2 \times 2 \times 3 \times 2 = 24$$

$$\frac{5}{6}=\frac{20}{24} \qquad \frac{3}{8}=\frac{9}{24} \qquad \frac{5}{12}=\frac{10}{24} \qquad \frac{1}{4}=\frac{6}{24}$$

$$\frac{20}{24}+\frac{9}{24}+\frac{10}{24}+\frac{6}{24}=\frac{45}{24}=\frac{15}{8}=1\frac{7}{8}$$

Add:

8. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$. **9.** $\frac{2}{3}+\frac{8}{15}+\frac{4}{30}$. **10.** $\frac{1}{8}+\frac{5}{12}+\frac{8}{18}$.

11. $\frac{3}{5}+\frac{1}{25}+\frac{7}{75}$. **12.** $\frac{3}{8}+\frac{7}{24}+\frac{5}{32}$. **13.** $\frac{1}{7}+\frac{3}{14}+\frac{11}{28}$.

14. $\frac{1}{2}+\frac{2}{3}+\frac{5}{6}+\frac{7}{12}$. **15.** $\frac{1}{5}+\frac{1}{3}+\frac{1}{15}+\frac{1}{45}$.

16. $\frac{3}{10}+\frac{7}{20}+\frac{9}{40}+\frac{7}{8}$. **17.** $\frac{8}{9}+\frac{25}{27}+\frac{7}{18}+\frac{5}{36}$.

18. $\frac{5}{11}+\frac{5}{33}+\frac{5}{22}+\frac{5}{8}$. **19.** $\frac{3}{7}+\frac{5}{9}+\frac{5}{12}+\frac{5}{72}$.

MIXED NUMBERS

20. Add: $2\frac{1}{2}$, $3\frac{1}{3}$, $4\frac{1}{4}$, and 5.

Whole numbers: $2+3+4+5=14$.

Fractions: $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{6}{12}+\frac{4}{12}+\frac{3}{12}=\frac{13}{12}=1\frac{1}{12}$.

Total: $14+1\frac{1}{12}=15\frac{1}{12}$.

21. Add: $3\frac{2}{3}$, 6. **22.** Add: \$$5\frac{1}{4}$, $66\frac{2}{3}$¢.

ADDITION OF FRACTIONS

When fractions have the same denominators, we can add them by adding their numerators.

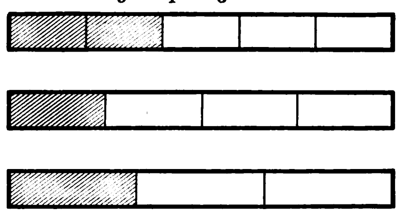
$$\tfrac{1}{4} + \tfrac{2}{4} + \tfrac{3}{4} + \tfrac{5}{4} = \tfrac{11}{4} \qquad \tfrac{11}{4} = 2\tfrac{3}{4}$$

We do not add the denominators because the denominators tell us only the number of fractional parts in the units; the size of these fractional parts remains the same.

Add : $\frac{2}{9}$, $\frac{3}{9}$, $\frac{7}{9}$, and $\frac{1}{9}$. $\qquad \frac{2}{9} + \frac{3}{9} + \frac{7}{9} + \frac{1}{9} = \frac{13}{9} = 1\frac{4}{9}$

But we cannot at once add fractions with different denominators.

$$\tfrac{2}{5} + \tfrac{1}{4} + \tfrac{1}{3} = ?$$



By these forms it is easy to see why it is impossible at once to add fractions that represent equal units divided into unequal parts.

If we divide these forms into more equal parts, that is, into smaller fractional parts, then we can add these fractions of the forms together. But into how small parts shall we divide them? The *least common multiple* tells us.

5, 4, and 3 have no common factors. $5 \times 4 \times 3 = 60$.

We must divide these forms into 60 equal parts.

$\frac{2}{5}$ = how many sixtieths?	$60 \div 5 = 12$	$12 \times 2 = 24$	$\frac{2}{5} = \frac{24}{60}$
$\frac{1}{4}$ = how many sixtieths?	$60 \div 4 = 15$	$15 \times 1 = 15$	$\frac{1}{4} = \frac{15}{60}$
$\frac{1}{3}$ = how many sixtieths?	$60 \div 3 = 20$	$20 \times 1 = 20$	$\frac{1}{3} = \frac{20}{60}$

$$\tfrac{24}{60} + \tfrac{15}{60} + \tfrac{20}{60} = \tfrac{59}{60}$$

Since 59 and 60 have no common factor, this fraction is at its lowest terms. No smaller denominator would be large enough to help us to add these fractions together.

REDUCING FRACTIONS

Sometimes we have such fractions as $\frac{2}{8}$, $\frac{6}{12}$, $\frac{4}{20}$, and prefer to have them changed to smaller numerators and denominators.

We know that $\frac{1}{4} = \frac{2}{8}$. Dividing both the numerator and the denominator of $\frac{2}{8}$ by 2 does not change its real value, because, though we make each equal part as shown by the denominator twice as large, we take only half as many equal parts. Whenever we change the sizes of denominator and numerator in such a way as not to change their ratio to each other, the value of the fraction is unchanged.

1. Reduce $\frac{120}{180}$ to its lowest terms.

$$10\left|\frac{120}{180}\right. = \frac{12}{18} \quad 2\left|\frac{12}{18}\right. = \frac{6}{9} \quad 3\left|\frac{6}{9}\right. = \frac{2}{3}$$

(the divisors 10, 2 and 3 are written beside both numerator and denominator)

Another way:

$$\begin{array}{r|rr} 10 & 120 & 180 \\ \hline 3 & 12 & 18 \\ \hline 2 & 4 & 6 \\ \hline & 2 & 3 \end{array}$$

greatest common divisor $10 \times 2 \times 3 = 60$

$$60\left|\frac{120}{180}\right. = \frac{2}{3}$$

We divide each of the terms of the fraction $\frac{120}{180}$ by 10, 2, and 3, factors common to both numerator and denominator, and obtain $\frac{2}{3}$. The numerator and denominator 2 and 3 are prime to each other. Hence the fraction $\frac{2}{3}$ is in its lowest terms.

Reduce to lowest terms these fractions:

2. $\frac{48}{96}$.	**3.** $\frac{75}{100}$.	**4.** $\frac{72}{100}$.	**5.** $\frac{18}{27}$.
6. $\frac{15}{25}$.	**7.** $\frac{21}{29}$.	**8.** $\frac{8}{36}$.	**9.** $\frac{12}{96}$.
10. $\frac{96}{144}$.	**11.** $\frac{72}{96}$.	**12.** $\frac{48}{54}$.	**13.** $\frac{63}{72}$.
14. $\frac{36}{108}$.	**15.** $\frac{24}{56}$.	**16.** $\frac{39}{69}$.	**17.** $\frac{24}{144}$.

SUBTRACTION OF FRACTIONS

1. From \$5 take $\frac{4}{5}$ of a dollar.

$$\$5 = \$4\tfrac{5}{5} \qquad 4\tfrac{5}{5} - \tfrac{4}{5} = \$4\tfrac{1}{5}$$

2. From $1\frac{3}{4}$ take $1\frac{1}{9}$.

$$1\tfrac{3}{4} = \tfrac{7}{4} \qquad 1\tfrac{1}{9} = \tfrac{10}{9} \qquad \tfrac{7}{4} - \tfrac{10}{9} = \tfrac{63}{36} - \tfrac{40}{36} = \tfrac{23}{36}$$

In this example we reduce the mixed numbers to improper fractions, and these fractions to a common denominator. Finding the difference between the numerators, we write it over the common denominator, and obtain the required difference, $\frac{23}{36}$.

3. From $7\frac{1}{3}$ take $6\frac{7}{8}$.

$$7\tfrac{1}{3} = 6 + \tfrac{3}{3} + \tfrac{1}{3} = 6\tfrac{4}{3} \qquad 6\tfrac{4}{3} - 6\tfrac{7}{8} = 6\tfrac{32}{24} - 6\tfrac{21}{24}$$

$$\begin{array}{r} 6\frac{32}{24} \\ 6\frac{21}{24} \\ \hline \frac{11}{24} \end{array}$$

4. Take $2\frac{3}{8}$ from $4\frac{1}{4}$.

$$4\tfrac{1}{4} = 4\tfrac{2}{8} \qquad 3\tfrac{10}{8} - 2\tfrac{3}{8} = 1\tfrac{7}{8}$$

We reduce the fractions to a common denominator. Then, as we cannot take $\frac{3}{8}$ from $\frac{2}{8}$, we add 1, taken from 4, or $\frac{8}{8}$ to the $\frac{2}{8}$, making $\frac{10}{8}$.

Another way is to reduce both mixed numbers to improper fractions.

$$4\tfrac{2}{8} = \tfrac{34}{8} \qquad 2\tfrac{3}{8} = \tfrac{19}{8} \qquad \tfrac{34}{8} - \tfrac{19}{8} = \tfrac{15}{8} \qquad \tfrac{15}{8} = 1\tfrac{7}{8}$$

Find the answers to these questions:

5. $2\frac{3}{8} - 1\frac{5}{8} = ?$ **6.** $22\frac{1}{3} - 21\frac{5}{9} = ?$ **7.** $5\frac{3}{14} - 3\frac{6}{7} = ?$

8. $20\frac{11}{28} - 4\frac{11}{21} = ?$ **9.** $5\frac{3}{14} - 3\frac{6}{7} = ?$ **10.** $20\frac{11}{28} - 4\frac{11}{21} = ?$

11. $5 - 3\frac{8}{13} = ?$ **12.** $200 - 1\frac{7}{9} = ?$ **13.** $27\frac{1}{3} - 26\frac{26}{27} = ?$

14. $8\frac{7}{24} - 7\frac{11}{12} = ?$ **15.** $11\frac{7}{33} - \frac{21}{22} = ?$ **16.** $1\frac{12}{60} - \frac{99}{100} = ?$

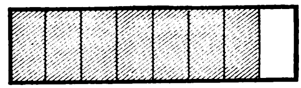
TELL

SUBTRACTION OF FRACTIONS

SAME DENOMINATOR

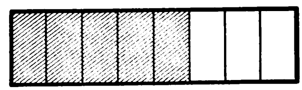
From $\frac{7}{8}$ take $\frac{5}{8}$.

$\frac{7}{8}$



$\frac{7}{8} - \frac{5}{8} = \frac{2}{8}$

$\frac{5}{8}$



Since both fractions are eighths, we need to find only the difference, 2, between the numerators, and to write it over the common denominator, 8.

1. From $\frac{7}{11}$ take $\frac{2}{11}$.

2. From $\frac{11}{19}$ take $\frac{7}{19}$.

3. From $\frac{27}{37}$ take $\frac{4}{37}$.

4. From $\frac{167}{711}$ take $\frac{19}{711}$.

5. From $\frac{7}{20}$ take $\frac{5}{20}$.

6. From $\frac{97}{100}$ take $\frac{17}{100}$.

DIFFERENT DENOMINATORS

WRITE

From $\frac{17}{18}$ take $\frac{7}{12}$.

Here the denominators are different, and we must first reduce the fractions to a common denominator.

$18 = 3 \times 3 \times 2$

$12 = 3 \times 2 \times 2$

Hence, the least common denominator is $3 \times 3 \times 2 \times 2 = 36$.

$\frac{17}{18} = \frac{34}{36}$ $\qquad \frac{7}{12} = \frac{21}{36}$ $\qquad \frac{34}{36} - \frac{21}{36} = \frac{13}{36}$

7. From $\frac{3}{4}$ take $\frac{1}{2}$. **8.** From $\frac{2}{7}$ take $\frac{4}{21}$. **9.** $\frac{3}{7} - \frac{4}{10} = ?$

Answer the following questions:

10. $\frac{5}{6} - \frac{2}{5} = ?$ **11.** $\frac{7}{11} - \frac{2}{7} = ?$ **12.** $\frac{2}{9} - \frac{1}{7} = ?$

13. $\frac{5}{13} - \frac{4}{11} = ?$ **14.** $\frac{17}{20} - \frac{4}{5} = ?$ **15.** $\frac{23}{24} - \frac{43}{48} = ?$

16. $\frac{3}{5} - \frac{1}{3} = ?$ **17.** $\frac{1}{2} - \frac{1}{5} = ?$ **18.** $\frac{11}{12} - \frac{19}{24} = ?$

19. $\frac{15}{16} - \frac{27}{32} = ?$ **20.** $\frac{9}{10} - \frac{17}{20} = ?$ **21.** $\frac{21}{40} - \frac{19}{80} = ?$

FRACTIONS

Like fractions indicate the same number of equal parts in the unit. Like fractions have the same denominator.

Unlike fractions have different denominators, which indicate different numbers of equal parts in the unit.

1. Reduce to improper fractions these mixed numbers:

$7\frac{1}{2}$, $15\frac{5}{8}$, $4\frac{7}{11}$, $102\frac{5}{9}$, $5\frac{9}{10}$, $19\frac{3}{4}$, $9\frac{19}{21}$, $236\frac{4}{19}$,
$10\frac{11}{12}$, $8\frac{9}{10}$, $19\frac{3}{4}$, $19\frac{19}{20}$, $28\frac{7}{10}$, $10\frac{1}{19}$, $215\frac{5}{14}$, $12\frac{3}{5}$,
$10\frac{11}{12}$, $15\frac{4}{27}$, $84\frac{3}{20}$, $20\frac{5}{11}$, $20\frac{9}{35}$, $25\frac{4}{6}$, $100\frac{7}{8}$, $132\frac{4}{25}$.

2. Reduce these improper fractions to mixed numbers:

$\frac{11}{4}$, $\frac{15}{3}$, $\frac{19}{5}$, $\frac{21}{6}$, $\frac{35}{7}$, $\frac{41}{6}$, $\frac{54}{9}$, $\frac{63}{7}$,
$\frac{839}{60}$, $\frac{31}{10}$, $\frac{72}{11}$, $\frac{95}{12}$, $\frac{108}{9}$, $\frac{121}{18}$, $\frac{111}{24}$, $\frac{216}{19}$,
$\frac{718}{44}$, $\frac{908}{60}$, $\frac{910}{57}$, $\frac{311}{47}$, $\frac{926}{58}$, $\frac{1011}{74}$, $\frac{2016}{98}$, $\frac{26}{7}$,
$\frac{92}{9}$, $\frac{94}{11}$, $\frac{2000}{73}$, $\frac{65}{14}$, $\frac{236}{17}$, $\frac{426}{13}$, $\frac{863}{12}$, $\frac{101}{17}$,
$\frac{100}{31}$, $\frac{112}{7}$, $\frac{117}{16}$, $\frac{289}{17}$, $\frac{200}{33}$, $\frac{110}{12}$, $\frac{201}{8}$, $\frac{50}{7}$.

We have practiced **factoring** and **cancellation,** have found **greatest common divisors** and **least common multiples,** and are studying **improper fractions,** and **mixed numbers** partly because we need to know these processes so as to add, to subtract, to multiply, and to divide fractions.

Tell the meaning of each of the expressions in **boldface** type.

These processes are not so important in themselves as are addition, subtraction, multiplication, and division of whole numbers, but without knowing them thoroughly we can never succeed in the later and higher subjects of Arithmetic.

ADDITION OF MIXED NUMBERS

Add : $31\frac{2}{5}$, $3\frac{5}{6}$, and $15\frac{3}{10}$.

$$\begin{array}{r} 31 \\ 3 \\ \underline{15} \\ 49 \\ \underline{1\frac{8}{15}} \\ 50\frac{8}{15} \end{array}$$

$\frac{2}{5} = \frac{12}{30}$. $\frac{5}{6} = \frac{25}{30}$. $\frac{3}{10} = \frac{9}{30}$.

$\frac{2}{5} + \frac{5}{6} + \frac{3}{10} = \frac{12}{30} + \frac{25}{30} + \frac{9}{30} = \frac{46}{30} = \frac{23}{15} = 1\frac{8}{15}$.

Add :

1. $5\frac{1}{9}$, $6\frac{8}{9}$, 7.
2. $3\frac{1}{5}$, $2\frac{4}{5}$.
3. $4\frac{1}{3}$, $5\frac{2}{5}$, $1\frac{4}{15}$.
4. $2\frac{2}{3}$, $4\frac{5}{6}$.
5. $2\frac{1}{5}$, $\frac{21}{15}$, $\frac{1}{9}$, 4.
6. $6\frac{1}{4}$, $5\frac{3}{8}$, $7\frac{7}{12}$, 11.
7. $1\frac{1}{2}$, $2\frac{3}{4}$, $3\frac{7}{8}$, $4\frac{7}{16}$.
8. $5\frac{5}{6}$, $4\frac{5}{9}$, $3\frac{5}{18}$, $\frac{5}{12}$.
9. $12\frac{1}{11}$, $1\frac{3}{22}$, $7\frac{5}{33}$.
10. $\frac{19}{3}$, $3\frac{1}{5}$, $\frac{5}{12}$, $\frac{7}{20}$.
11. $1\frac{3}{14}$, $\frac{60}{49}$, $\frac{11}{7}$, 2.
12. $11\frac{1}{18}$, $2\frac{7}{27}$, $\frac{1}{54}$, $\frac{2}{9}$.
13. $3\frac{2}{7}$, $5\frac{8}{63}$, $3\frac{11}{21}$, 2.
14. $4\frac{1}{7}$, $4\frac{1}{14}$, $4\frac{1}{28}$, $\frac{1}{49}$.
15. $\frac{27}{5}$, $\frac{48}{25}$, $\frac{101}{15}$.
16. $\frac{41}{22}$, $1\frac{5}{33}$, $1\frac{5}{6}$.
17. $\frac{99}{40}$, $7\frac{9}{25}$, $8\frac{7}{50}$.
18. $\frac{1000}{24}$, $\frac{100}{8}$, $\frac{10}{3}$.
19. $\frac{63}{7}$, $\frac{45}{14}$, $\frac{100}{28}$.
20. $\frac{300}{21}$, $\frac{400}{49}$, $\frac{5}{3}$.
21. $2\frac{3}{5}$, $5\frac{1}{10}$, $2\frac{11}{20}$.
22. $1\frac{1}{4}$, $4\frac{7}{12}$, $3\frac{1}{16}$, $\frac{27}{24}$.
23. 2, $3\frac{4}{11}$, $\frac{231}{55}$, $\frac{491}{33}$.
24. $1\frac{1}{9}$, $1\frac{1}{30}$, $1\frac{1}{15}$, $1\frac{1}{45}$.
25. 4, $5\frac{1}{9}$, $\frac{271}{12}$, $\frac{88}{45}$.
26. $2\frac{1}{2}$, $3\frac{3}{4}$, $4\frac{7}{8}$.
27. $2\frac{5}{7} + \frac{3}{7} + 4\frac{9}{14} + 8$.
28. $3\frac{3}{7} + 4\frac{11}{14} + 5\frac{23}{28}$.
29. $3\frac{1}{5} + 2\frac{3}{10} + 5\frac{11}{15} + 6\frac{7}{20}$.
30. $\frac{7}{18} + 2\frac{1}{4} + 8 + 8\frac{5}{6}$.
31. $13\frac{1}{9} + 5\frac{3}{10} + \frac{11}{45}$.
32. $\frac{1}{3} + 2\frac{1}{5} + 7\frac{1}{9} + 4\frac{5}{12}$.
33. $3\frac{2}{3} + 4\frac{2}{5} + \frac{1}{15}$.
34. $4\frac{1}{9} + 7\frac{1}{5} + 3\frac{11}{45}$.
35. $2\frac{1}{8} + 7\frac{4}{5} + 2\frac{3}{10}$.
36. $\frac{2}{3} + \frac{8}{9} + 2\frac{1}{2} + 3$.

REVIEW

1. If $34,051 be divided into 12 equal parts, how many dollars will there be in one of the parts?

2. A box contained 1128 eggs. How many dozen eggs were there in the box?

3. A grocer bought 1416 eggs at 9¢ a dozen. How much did they cost him?

4. If 7 dozen eggs cost 168¢, what was the price a dozen? What was the cost of each egg?

5. The wages of 13 men on equal pay for one week amounted to $97.50. How much did each earn a day?

6. The expense of building a bridge was $8743, and that of opening a road was $2163. The total expenses were borne equally by seven towns. How much did each town pay?

7. A man receives a salary of $1200 a year. Out of this he saves $212 each year. How much does he spend a week on the average?

8. A contractor requires one million bricks. He has on hand 559,941 bricks. How many loads of 437 bricks each does he need to make up the full number?

9. A bushel of wheat weighs 60 pounds, and a bushel of oats weighs 34 pounds. How many bushels of oats will weigh as much as 187 bushels of wheat?

10. There are how many feet in a mile? How many steps, each 2 feet long, would you take in walking a mile?

11. Seven men have each an equal interest in a farm of 107 acres. They sell it at $56 an acre. How many dollars should each receive?

QUESTIONS

Many practical problems that men have to solve can be solved only when we understand **equivalence of fractions, reducing fractions to lowest terms,** finding **least common multiple,** and addition and subtraction of fractions. Be ready to tell orally the meaning of each expression in **bold-face** type.

1. Mr. Smith owns $\frac{3}{8}$ of a section of land, Mr. Jones owns $\frac{5}{16}$ of the section, and Mr. Brown owns the remainder of it. What fraction of the section does Mr. Brown own?

2. A section of land contains 640 acres. How many acres has Mr. Smith in his part of the section? How many has Mr. Jones in his?

3. An acre contains how many square feet? How many square feet are there in $\frac{1}{4}$ of an acre? $\frac{3}{8}$? $1\frac{1}{3}$?

4. A lot contains a half acre of land. It is four times as large as the next lot of land. How many square feet does the second lot contain?

5. A man expended $\frac{1}{3}$ of his salary for board, $\frac{1}{5}$ of it for clothing, $\frac{1}{8}$ of it for books, and $\frac{3}{10}$ of it for other purposes. What part of his salary had he then left?

6. What number must be added to the sum of $1\frac{7}{20}$ and $2\frac{2}{3}$ to make $26\frac{1}{4}$?

7. Charles paid a debt of \$$7\frac{2}{5}$ out of a purse containing \$$9\frac{1}{12}$. How many dollars remained?

8. A person who had $13\frac{5}{8}$ yards of cloth sold $7\frac{3}{4}$ yards. How many yards remained?

9. A milkman sold $33\frac{3}{4}$ gal. of milk from $36\frac{1}{2}$ gal. How many gallons remained? How many quarts?

REVIEW

1. A, B, C, and D built a mill. A invested $7418; B invested $9475; C, $8643; and D, $7464. How much did the mill cost the builders?

2. In a cattle train there were two cars with 17 head of cattle in each, three cars with 19 head in each, one car with 22 head, and two cars with 21 head in each. How many cattle were there in all the cars?

3. A farmer has 27 A. of land under wheat, 15 A. under oats, 14 A. in meadow, 19 A. in pasture, 9 A. under peas, 6 A. in potatoes, 7 A. in turnips, 19 A. under corn, 5 A. in orchard, 3 A. under garden vegetables, and 20 A. of woods. How many acres are there in his farm?

4. A man bought three farms. For the first farm he paid $4793 for the land, $479 for the crop, and $698 for the live stock; for the second he paid $5986 for the land, $973 for the crop, and $546 for the live stock; for the third he paid $8678 for the land, $1094 for the crop, and $783 for the live stock. What was the total amount paid?

5. Franklin was born in 1706 and died in 1790. Washington was born in 1732 and died in 1799. How old was each when he died?

6. Our Declaration of Independence was adopted in 1776. The landing of the Pilgrims took place 156 years before the Declaration was adopted, and it is said that gunpowder was invented in Europe 200 years before the landing of the Pilgrims. Give the supposed year of the invention of gunpowder in Europe.

7. France contains 204,092 square miles. Texas contains 265,780 square miles. How many more square miles are there in Texas than in France?

MONEY

What is the smallest coin we use?

What is the value of a nickel? dime? quarter?

We write seven dollars sixty-three cents, \$7.63; twenty-one dollars fourteen cents, \$21.14; two dollars four cents, \$2.04; eleven dollars ninety cents, \$11.90.

$\frac{1}{10}$ of a cent is called a mill. Mills are written at the right of cents: we write three cents three mills, \$.033; one dollar sixteen cents nine mills, \$1.169; twenty dollars five mills, \$20.005.

10 mills make 1 cent

100 cents make 1 dollar

Mills occupy the third place at the right of the decimal point, and cents the first two places.

The United States government issues gold coins.

quarter eagle	= \$2.50	eagle	= \$10.00
half-eagle	= \$5.00	double eagle	= \$20.00

What bills are issued by government? What was the value of the largest bill you have ever seen? What was the value of the smallest?

1. What three coins together make 25¢?

2. What coins together make 11¢? 16¢? 19¢? 31¢? 49¢? 51¢? 65¢? 75¢?

3. How many nickels would you give for \$$\frac{1}{2}$? How many dimes? How many quarters?

4. What is the smallest silver coin called? What is the value of our largest silver coin?

5. How many mills are there in a half dollar?

6. Silver is worth about 56¢ an ounce. A silver dollar weighs $\frac{7}{8}$ of an ounce. What is the value of the silver in a dollar?

MONEY

$\$\frac{1}{4}$ expressed decimally is \$.25. $\$\frac{1}{2}$ = how many cents?

1. Write decimally $\$\frac{1}{2}$, $\$\frac{3}{4}$, $\$\frac{1}{5}$, $\$\frac{7}{10}$, $\$\frac{1}{20}$.

2. Read and add:

\$ 1.46	\$ 19.01	\$ 77.77	\$ .05
2.00	19.011	10.01	.005
20.00	60.03	44.04	.50
200.00	113.13	266.666	50.051
5.00	200.20	1.001	1.

3. Is there any difference between $\$1\frac{1}{2}$ and \$1.50?

4. Is there any difference between $\$\frac{1}{10}$ and 1 dime?

5. Write eight dollars two dimes and two cents.

6. How many dollars are there in \$39.468? How many dimes or *tenths* of a dollar? How many cents or hundredths of a dollar? How many mills or thousandths?

7. Write fifty cents in two ways.

8. How many dollars and cents are there in:

330¢?	275¢?	122¢?	196¢?	288¢?	110¢?
108¢?	222¢?	770¢?	704¢?	636¢?	565¢?

9. Write decimally:

6 dollars 2 dimes 8 cents 8 mills
22 dollars 7 dimes 3 cents 1 mill
303 dollars 0 dimes 1 cent 0 mills
72 dollars 6 dimes 0 cents 0 mills
66 dollars 6 dimes 6 cents 6 mills

10. How many $\$\frac{1}{2}$ are there in \$6? \$20? \$22? \$30?

11. How many \$.25 are there in \$1? \$2? \$4? \$8? \$10?

12. How many cents are there in two half dollars? in three quarters?

MONEY

	1.	2.	3.	4.
Add:	$ 432.28	$ 1.	$ 400.89	$ 93.80
	92.01	8.72	398.71	110.
	86.	15.75	.43	364.82
	1491.	18.61	109.61	943.81
	80291.69	92.	83.40	89.62
	3432.86	100.	1298.60	498.

1 represents the chief items in the cost of a schoolhouse, with twenty-four rooms and an assembly hall; **2** represents the items of the cost of a railroad journey from Boston to Denver, one way; **3** represents the cost of the stock of a small store of general merchandise; and **4** represents the money received by a farmer for the chief products of a 320-acre farm in one year.

5. A man who had saved $10,000 decided to build a house. He estimated its cost in these items: land, $2000; contract for house, $5500; architect's plans and services, $300; heating plant, $500; grading the grounds, $100; furniture, $1000. When his whole house was finished and his family had moved in, he found that the total cost had been $\frac{1}{5}$ more than his estimate. How much more was the cost than the amount he intended to spend?

Subtract

6.	7.	8.
$864.92	$1921.92	$7284391.82
776.15	873.86	5000000.00

9. Multiply six hundred ninety-one thousand four hundred forty-five dollars seventy cents by thirty-two, and prove the result by division.

10. At $2.50 a day what does a man earn in 300 days?

MONEY AND TAXES

Mills are not coined. But they have very important duties to perform. All the people must help support the **government**, which rules **our country** for us. We have to pay for the government buildings, and for the services of those engaged in government. We raise the money for all government purposes by **taxes** upon the wealth of the citizens. The wealth that people own themselves is called **property**. We usually count taxes as so many dollars, cents, and mills upon every hundred dollars' worth of property.

One State allows towns and cities to tax every $100 worth of property three mills to support a public library. A man with a house worth $3100 would have to pay 31 times 3 mills or 93 mills a year for the support of the public library, if his town had such a public library. 93 mills = 9.3¢. Mills help us to arrange for very small amounts of tax from a great many different people; yet all these amounts together make a great deal of money.

A town with a 3-mill tax for a public library would get $3000 a year from $10,000,000 of property.

COINAGE

All the metal money of the United States is coined in the Government Mints. Of these Mints the largest is in the city of Philadelphia. Do you know where Philadelphia is? Do you know anything about Philadelphia? What do you know about our Mints?

1. How many cents or hundredths of a dollar are there in $1.025? How many mills or thousandths of a dollar?
2. What is the difference between $2½ and 75¢?

LIQUID MEASURE

Liquid Measure is used in measuring all kinds of liquids.

TABLE

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

In measuring the capacity of reservoirs, tanks, cisterns, etc., $31\frac{1}{2}$ gal. make a barrel (bbl.), and 63 gal. a hogshead (hhd.), but in commerce the barrel and the hogshead vary in capacity. The standard unit of liquid measure is the gallon. It contains 231 cu. in.

WRITE

1. How many quarts are there in 2 gal.? 3 gal.? 4 gal.?
2. How many pints are there in 2 qt.? 3 qt.? 5 qt.?
3. How many gills are there in 2 pt.? 3 pt.? 6 pt.?
4. How many quarts are there in 16 pt.? 24 pt.? 21 pt.?
5. How many gallons are there in 28 qt.? 32 qt.? 25 qt.? 36 qt.? 40 pt.?
6. How many quarts are there in $\frac{1}{2}$ gal.? $\frac{7}{8}$ gal.?
7. What is the value of a can of milk containing $66\frac{2}{3}$ pt. at 6¢ a quart?
8. A grocer sold in one month 8 bbl. of oil at 50¢ a gallon. What was the total amount of money he received?
9. He bought 3 hhd. of molasses at $24 a hhd. and sold the molasses at 50¢ a gal. What did he gain?
10. A reservoir holds 30 000 000 bbl. of water. At 10 gal. a day each how long would this last 1000 people?

DRY MEASURE

Dry Measure is used in measuring grain, fruits, and vegetables.

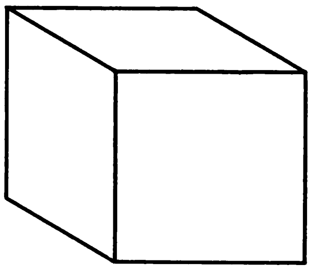
TABLE

2 pints (pt.)	=	1 quart (qt.)
8 quarts	=	1 peck (pk.)
4 pecks	=	1 bushel (bu.)
$2\frac{1}{2}$ bushels	=	1 barrel (bbl.)

WRITE

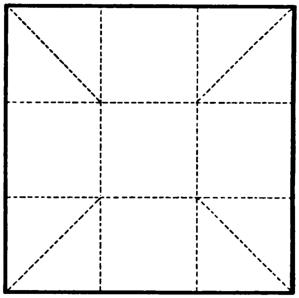
1. How many pecks are there in 2 bu.? 4 bu.? 5 bu.?
2. How many quarts are there in 2 pk.? 3 pk.? 6 pk.?
3. How many pints are there in 2 qt.? 7 qt.? 9 qt.?
4. How many pints are there in 1 bu.? 2 bu.?
5. How many quarts are there in $\frac{1}{2}$ bu.? $\frac{1}{4}$ bu.? $\frac{3}{4}$ bu.?
6. How many pecks are there in $\frac{1}{4}$ bu.? $\frac{1}{2}$ bu.? $1\frac{3}{4}$ bu.?
7. How many pecks are there in 23 qt.? 41 qt.?
8. How many bushels are there in 8 pk.? 17 pk.? 27 pk.? 32 pk.?
9. John picked 64 pk. of strawberries at 3¢ a quart. His employer sold them at 8¢ a quart. How much did John receive? What did his employer gain, if his other expenses were not counted?
10. A housekeeper bought 2 bu. of potatoes and used them at the rate of 3 pt. a day. How long did they last?
11. An apple orchard bore 100 bu. of apples. The farmer sold $\frac{1}{2}$ of them at $1 a barrel. How many dollars did he receive for these apples?
12. A family used 2 bbl. of apples in one month. How many pt. was this per day?

MEASURES OF CAPACITY



Cubic inch

$1\frac{1}{2} \times 1\frac{1}{2}$



$1\frac{1}{2}$ in. $\times$ $1\frac{1}{2}$ in. ($\frac{1}{2}$ size)

4 gills make 1 pint.

1. Take a liquid gill measure and find how many cubic inches it contains.

A cubic inch may be made out of cardboard or heavy paper. Fold at the lines dotted. This will hold water, but dry sand may be used.

A cardboard box of the volume given by sides 3 in. $\times$ 7 in. $\times$ 11 in. will be found to contain almost exactly 1 gallon.

2. Make the box of paper 3 in. $\times$ 3 in.

3. Find the number of cubic inches in a dry quart.

4. Find the contents in cubic inches of a drinking glass. Compare this with a pint.

5. What is the capacity in cubic feet of a distributing city reservoir 20 ft. $\times$ 500 ft. $\times$ 3000 ft.?

6. What is the volume of a square prism 3 in. by 3 in. by 2 ft.? of a prism 5 ft. by 2 ft. by 2 ft.?

AVOIRDUPOIS WEIGHT

Avoirdupois weight is used for weighing all articles except gold, silver, jewels, and medicines when compounded.

TABLE

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight, 2000 pounds	= 1 ton (T.)

WRITE

1. How many pounds are there in 2 T.? 3 T.? 4 T.? 10 T.?
2. How many pounds are there in 2 cwt.? 3 cwt.? 5 cwt.? 8 cwt.?
3. How many ounces are there in 2 lb.? 3 lb.? 5 lb.?
4. How many tons are there in 6000 lb.? in 8000 lb.? in 20,000 lb.?
5. How many ounces are there in $\frac{1}{2}$ lb.? in $1\frac{1}{2}$ lb.?
6. How many ounces are there in 1 cwt.?
7. John bought 8 oz. of soda. What part of a pound was that?
8. He bought also $\frac{1}{2}$ ton of coal. How many pounds was that? If his mother used two buckets of coal a day, and each bucket weighed 20 pounds, how many days did the coal last?
9. A hay wagon weighed 1400 pounds, and the hay on it weighed 3600 pounds. The two horses weighed each 1100 pounds, while two men on the wagon weighed each 150 pounds. Was it safe to cross a bridge built to support 3 tons?

A "long ton" used for coal at the mines and for merchandise at the United States Custom Houses weighs 2240 pounds.

MEASURE OF TIME

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 year (yr.)
366 days	= 1 leap year (yr.)
100 years	= 1 century (C.)

Leap year comes every four years, when February gains a day.

The year is divided into 12 calendar months:

January (Jan.)	31 da.	July	31 da.
February (Feb.)	28 da.	August (Aug.)	31 da.
March (Mar.)	31 da.	September (Sept.)	30 da.
April (Apr.)	30 da.	October (Oct.)	31 da.
May	31 da.	November (Nov.)	30 da.
June	30 da.	December (Dec.)	31 da.

Thirty days hath September,
April, June, and November,
All the rest have thirty-one,
Excepting February alone.
Twenty-eight are all its store
Till leap year gives it one day more.

Until the year 2400 every year we can divide by 4 will be leap year. We usually call thirty days a month unless we know the exact month in question.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

This calendar is true for any month when the first day of the month falls on Sunday and when the month has 31 days. This calendar represents December, 1901, and March, 1903. If the 31st day were omitted, it would represent June, 1902, and November, 1903, also.

COUNTING

In counting many kinds of articles we use these measures:

12 things	= 1 dozen (doz.)
12 dozen	= 1 gross (gr.)
12 gross	= 1 great gross (G. gr.)
20 things	= 1 score (sc.)

In buying and selling paper we use these measures:

24 sheets	= 1 quire (qr.)
20 quires	= 1 ream (R.)
2 reams	= 1 bundle (bdl.)
5 bundles	= 1 bale.

Reams of 500 sheets are now very common.

WRITE

1. How many lead pencils are there in $\frac{1}{4}$ gross?

2. How many sheets of paper are there in a ream?

3. How many quires are there in 2 R.? in 5 R.?

4. How many score are there in 100?

5. Name three kinds of articles sold by the dozen.

6. A school used 1 great gross of pens every term. How many single pens were used in three terms?

7. In a business office 4 reams of stationery were used in one month. How many sheets were used?

8. A daily newspaper in a great city used 120 bales of paper a day. How many sheets were used?

9. In her correspondence Mrs. Somers used in one year 25 quires of stationery. How many sheets was this?

10. How many things make 4 G. gr.?

STUDY 112

BILLS

A **bill of goods** is a written account of articles sold or of services rendered, with dates, place, prices, and names of creditor and debtor; that is, of the person to whom the money is due and of the person who owes the money.

New York, Aug. 1, 1901.

William Estabrook,

1291 Fifth Ave., New York.

Bought of Norton & Wright.

July	5	1/2 yd. Flannel	.50		25		
		pr. Rubbers			70		
		pr. Shoes		2	50		
		Tie		1			
		Basket			98		
	10	30 yd. Carpet	1.25	37	50		
	15	2 Shirts	.98 .69	1	67		
	24	10 yd. Silk	2.35	23	50	68	10
	10	By Cash				30	
		Balance due				38	10

Received payment

Norton & Wright.

Aug 5, 1901.

BILLS

Make out the following bills:

1. Mrs. C. Allen bought of Langdon & Co., Brooklyn, Oct. 25, 1900, 14 yd. print at 13¢ per yd; Nov. 3, 33 yd. cotton at 14¢; Nov. 6, 17 yd. tweed at $1.18; Nov. 12, 16 yd. silk at $1.87; 9 yd. lining at 13¢; 3 doz. buttons at 23¢; Nov. 14, 9 yd. jersey cloth at 45¢; 2 yd. plush at $1.95; 3 yd. lining at 18¢; 2 doz. buttons at 15¢.

2. Mrs. M. A. Taylor bought of Race & Co., Rochester, N. Y., Sept. 27, 1900, 9 lb. roast beef at 12¢; Nov. 21, 7 lb. lamb at 13¢; Nov. 23, 8 lb. beef at 8¢; Nov. 25, 3 lb. steak at 14¢; Nov. 26, 13 lb. lamb at 13¢; Nov. 28, 5 lb. corned beef at 9¢, and 2 geese at 65¢ each; Dec. 1, 3 chickens at 55¢, and 9 lb. sausages at 12¢.

3. Walter Hays, Omaha, sold to Harvey Richards, on Aug. 20, 1900, 14 cords maple at $8.50, 4 cords soft wood at $3.25, and 7 cords beech at $7.25.

4. George Pinkham bought of Horace Shaw, Kansas City, Sept. 11, 1900, 700 lb. of rye at $2.75 per cwt.; 400 lb. oatmeal at $2.25; 300 lb. cornmeal at $2.25; and 200 lb. buckwheat flour at $2.50. On Sept. 28, 1900, George Pinkham paid $25 on the above account.

5. C. L. Cook bought of Williams & Co., Louisville, April 16, 1900, 100 ft. $\frac{3}{4}$ in. rubber hose at 20¢ per ft.; 2 pr. $\frac{3}{4}$ in. couplings and fittings at 50¢ per pr.; 1 hose pipe, at $1.25; 3 step-ladders at $1.50.

6. Williams & Jones, New York, sold to R. M. Marston, Dec. 5, 1899, 19 yd. calico at 7¢; 17 yd. linen at 47¢; 16 yd. lining at 9¢; Dec. 21, 8 yd. flannel at 48¢; 23 yd. braid at 3¢; Dec. 26, 7 pr. stockings at 25¢; and 3 pr. gloves at 65¢. Paid in full, Jan. 2, 1900.

CANCELLATION

We can shorten division by dividing out the factors common to both dividend and divisor.

1. Divide 450 by 150.

$$\begin{array}{ccc} 45 & & 15 \\ \cancel{450} & \div & \cancel{150} \end{array} \quad \text{This takes out 10.}$$

$$\begin{array}{ccc} 9 & & 3 \\ \cancel{45} & \div & \cancel{15} \end{array} \quad \text{This takes out 5.}$$

$$\begin{array}{ccc} 3 & & 1 \\ \cancel{9} & \div & \cancel{3} \end{array} \quad \text{This takes out 3.}$$

$3 \div 1 = 3$, the quotient of $450 \div 150$

Write in this way:

$$\begin{array}{ccc} 3 & & 1 \\ \cancel{9} & & \cancel{3} \\ \cancel{45} & & \cancel{15} \\ \cancel{450} & \div & \cancel{150} \end{array} \quad 3 \div 1 = 3$$

2. Explain this:

$$\begin{array}{ccc} 8 & & 1 \\ \cancel{40} & & \cancel{5} \\ \cancel{200} & & \cancel{25} \\ \cancel{1000} & \div & \cancel{125} \end{array} \quad 8 \div 1 = 8$$

Removing common factors in division is **cancellation.**

3. What is the quotient of $45 \times 12 \times 16 \times 15$ divided by $9 \times 24 \times 5 \times 10$?

$$\begin{array}{cccc} 1 & & & \\ \cancel{3} & 1 & 1 & \\ \cancel{15} & \cancel{2} & \cancel{3} & \\ \cancel{45} \times & \cancel{16} \times & \cancel{15} \times & 12 \\ \hline \cancel{24} \times & \cancel{10} \times & \cancel{9} \times & \cancel{5} \\ \cancel{8} & \cancel{2} & \cancel{3} & 1 \\ 1 & 1 & & \end{array} = \frac{1 \times 1 \times 1 \times 12}{1 \times 1 \times 1 \times 1} = \frac{12}{1} = 12.$$

4. Divide 22×15 by 11×5.

5. Divide $24 \times 16 \times 25 \times 20$ by $10 \times 4 \times 6 \times 5 \times 40$.

Arrange the numbers in order of their size.

Be very careful to cancel only once each in dividend and divisor for any one common factor.

CANCELLATION

Divide: 1. 25×11 by 5×11.

2. 56×24 by 7×48.

3. $4 \times 6 \times 12$ by $3 \times 8 \times 4$.

4. $3 \times 50 \times 8$ by $4 \times 5 \times 2 \times 10$.

5. $5 \times 16 \times 4$ by 8×20.

6. $28 \times 33 \times 35$ by $15 \times 11 \times 14$.

7. $\frac{8 \times 7 \times 81}{27 \times 28}$.

8. $\frac{112 \times 24 \times 14}{12 \times 28 \times 16}$.

9. $\frac{36 \times 10 \times 7}{14 \times 5 \times 9}$.

10. $\frac{35 \times 33 \times 28}{14 \times 15 \times 11}$.

11. $\frac{21 \times 8 \times 40}{4 \times 7 \times 20}$.

12. $\frac{17 \times 64 \times 11 \times 90}{30 \times 22 \times 51 \times 8}$.

13. A boy bought 6 dozen eggs at 15¢ a dozen, and raised 4 dozen chickens, which he sold at 10¢ each. What was the ratio of the first cost to the returns?

14. There were 900 private soldiers in a regiment, who received $12 a month each. The officers received in all $1080 a month. What was the ratio of the total wages of the privates to the wages of the officers?

15. The salaries of the teachers of 10,000 children a year cost the city of W—— an average of $24 per child. The salaries of 300 policemen averaged $800 a year each. What was the ratio of the total cost of free public teaching to the cost of the free public police service?

Do you notice how easy it is to cancel 10's, 100's, and 1000's?

Arithmetic.

Room 14. Grade IV. William Estabrook.
School No. 5. March 25, 1901.

1. Divide 22 x 15 by 11 x 5.

$$\frac{\overset{2}{\cancel{22}} \times \overset{3}{\cancel{15}}}{\underset{1}{\cancel{11}} \times \underset{1}{\cancel{5}}} - \frac{2 \times 3}{1 \times 1} - \frac{6}{1} - 6$$

Answer, 6.

2. Add 10.5, .003, 15.641 and 300; and write the answer in words.

10.5
.003
15.641
300
326.144

Answer,
Three hundred twenty-six, and one hundred forty-four thousandths

3. $x - \frac{\$30 \times 2}{4}$ Find x.

$$x - \frac{\overset{15}{\$\cancel{30}} \times \cancel{2}}{\underset{\underset{1}{\cancel{2}}}{\cancel{4}}} \qquad x - \$15$$

Answer, $15

or

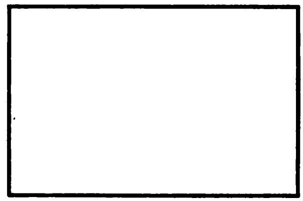
$$x - \frac{\$30 \times 2}{4} \qquad 4x - \$30 \times 2$$

$$4x - \$60 \qquad x - \$15$$

Answer, $15

AREAS

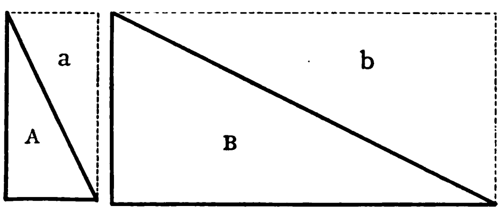
We find the **areas** of rectangles in square measure by multiplying the lengths of the adjoining sides.



1. If the sides of this rectangle were 4 inches and 6 inches, its area would be 4 sq. in. × 6 = 24 square inches = 24 sq. in.

2. If a mirror is 3 ft. × 4½ ft. in size, its area is 13½ sq. ft.

We find the areas of right-angled triangles by multiplying the lengths of the sides which make the right angle and dividing the result by two.



1 in. × ½ in. 1 in. × 2 in.

The dotted lines show the rectangle that the multiplication of the lengths of the two sides gives us.

3. Measure the triangle A, and find its area upon this scale. Length 1 in. = 4 ft. Width ½ in. = 2 ft. 4 sq. ft. × 2 = 8 sq. ft. 8 sq. ft. ÷ 2 = 4 sq. ft.

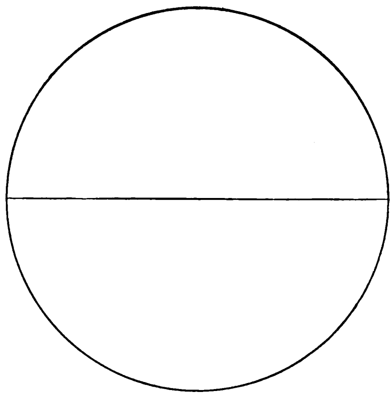
Or by cancellation

$$\frac{4 \text{ sq. ft.} \times \overset{1}{\cancel{2}}}{\cancel{2}} = 4 \text{ sq. ft.}$$

4. By the same scale find the area of the triangle B.

5. Draw various triangles on the blackboard. Measure them. Find their areas, using no fractions smaller than one fourth.

CIRCUMFERENCES



A circle may be drawn on the blackboard by making a loose knot in a string and setting the knot around the crayon. Then if one holds the free end of the string against the blackboard with one finger of the left hand, and stretches it tight, a circle of any desired diameter may be made. The finger marks the center. With a pin, a pencil, and a piece of cardboard with holes in it for the pin and pencil, circles may be drawn on paper. Circles may also be drawn with dividers or compasses.

The diameter of a circle is twice its radius. A radius is any straight line from the center to the circumference. The string makes the length of the radius of the circle drawn on the blackboard.

The circumference of any circle equals almost exactly three and a seventh times the diameter. We can prove this by drawing circles and comparing their diameters and circumferences.

1. Find the circumference of a circle 2 in. in diameter.

2 in. $\times 3\frac{1}{7} = 6\frac{2}{7}$ in.

2. Find the circumference of a circle 4 yd. in diameter.

4 yd. $\times 3\frac{1}{7} = 12\frac{4}{7}$ yd.

3. Draw circles of various diameters and find their circumferences.

In these questions we always need to know how to multiply a whole number and a fraction.

DECIMALS

We found that we could write five dollars and twenty-eight cents \$5.28. We called the period or point between 5 and 2 the **decimal point**. Decimal means ten or tenth. In our notation we used these orders.

Hundreds	Tens	Units
6	3	9

We can extend decimals to the fractions, tenth and hundredth, by the use of the decimal point.

Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths
6	3	9	.	4	2

39.4 is read thirty-nine four tenths. $39.4 = 39\frac{4}{10}$.

39.42 is read thirty-nine forty-two hundredths.

The whole number may be written $639\frac{42}{100}$.

1. Write in decimals $756\frac{84}{100}$; $\$15\frac{65}{100}$; $56\frac{1}{10}$ yd.

2. Read 15.3%; 2.7 hr.; 9.3 mo.

3. I bought 7.4 oz. of a very expensive kind of tea especially imported from China. I paid 10¢ an ounce. What was the cost?

Multiplying decimals by tens or hundreds is very easy.

$7.4¢ \times 10 = [7 \times 10]$ and $[\frac{4}{10}¢ \times 10] = 70¢ + 4¢ = 74¢$.

We can multiply a decimal by ten simply by moving the decimal point one place to the right, as you see.

4. If I had paid 20¢, what would have been the cost?

$20¢ = 10¢ \times 2$. $7.4¢ \times 10 = 74¢$. $74¢ \times 2 = 148¢$. $148¢ = \$1.48$.

Do you see that we can change cents to dollars by moving the decimal point 2 places to the left?

$100¢ = \$1$. $148¢ \div 100 = \$1.48$.

DECIMALS

We count United States money by decimals, which means by tens and tenths. Our system of counting is a decimal system. \$20.35 means two tens (twenty) dollars thirty (three tens) five cents (hundredths). The period which separates dollars and cents is a **decimal point.** To the left are units and tens; to the right are tenths and hundredths. The third figure to the left above tens shows hundreds; the third to the right shows thousandths.

We usually think of the decimal system in connection with fractions. $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{7}{10}$, $\frac{8}{100}$, and $\frac{2}{333}$ are **common fractions.** .1, .5, .8, .15, are **decimal fractions.** We read .1, one tenth; .5, five tenths; .8, eight tenths; and .15, fifteen hundredths.

Decimal Numeration Table

Thousands	Hundreds	Tens	Units	**Decimal Point**	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
3	4	7	1	.	6	5	9	8	7	2

We read the number here three thousand, four hundred seventy-one, and six hundred fifty-nine thousand, eight hundred seventy-two millionths.

The whole number is numerated from the *point* toward the left, and the decimal from the *point* toward the right.

Tenths, hundredths, etc., are called **places,** or **orders.**

In reading a decimal, we read it as a whole number and give the name of the right-hand figure only; thus, .23 is read, twenty-three hundredths. .246 is read, two hundred forty-six thousandths. .3429 is read, three thousand four hundred twenty-nine ten-thousandths.

DECIMALS

We need to learn accurately the value of each **place** in the decimal notation of fractions, just as we have already learned the decimal notation of whole numbers.

5.	five.	.5	five tenths.
50.	fifty.	.05	five hundredths.
500.	five hundred.	.005	five thousandths.
5000.	five thousand.	.0005	five ten-thousandths.
50000.	fifty thousand.	.00005	five hundred-thousandths.
500000.	five hundred thousand.	.000005	five millionths.
5000000.	five million.	.0000005	five ten-millionths.

Read:

1. 5.5. **2.** 7.8. **3.** 7.85. **4.** 8.365.
5. 569.5. **6.** 4002.8 **7.** 5.292. **8.** 51.3246.
9. 42879.48. **10.** 2.32982. **11.** .24318. **12.** 2.48295.

In reading decimals the ciphers need to be noticed very carefully. .205 is read, two hundred five thousandths. .025 is read, twenty-five thousandths. .00205 is read, two hundred five hundred-thousandths.

MEANING OF DECIMAL FRACTIONS

$.7 = \frac{7}{10}$. $.07 = \frac{7}{100}$. $.007 = \frac{7}{1000}$. $.0007 = \frac{7}{10000}$.
$2.7 = 2\frac{7}{10}$. $2.000007 = 2\frac{7}{1000000}$. $200.007 = 200\frac{7}{1000}$.

Change to common fractions: WRITE

13. .15. **14.** .75. **15.** 2.831. **16.** 14.9437.
17. 205.8. **18.** 2.0072. **19.** 90.98. **20.** 155.55.

The decimal fraction .88 means that there are one hundred equal parts and that eighty-eight parts are taken.

21. Tell the meaning of .482, .7254, and .325.

ADDING DECIMALS

It is much easier to add decimals than common fractions, because decimals have common denominators, such as tenths, hundredths, thousandths.

1. Add .25, .339, and .843.

We write these keeping the decimal points and the figures showing decimals of the same order under each other.

```
 .25
 .339
 .843
1.432
```

9 + 3 thousandths = 12 thousandths.

Set down 2 and carry ten thousandths which equal one hundredth.

1 + 4 + 3 + 5 hundredths = 13 hundredths.

Set down 3 and carry ten hundredths which equal one tenth.

1 + 8 + 3 + 2 tenths = 14 tenths which equal 1 and 4 tenths. We write one with the decimal point at its right.

2. Add: 4.125, 5609.68, 3.6045, 7.25.

Here we have whole numbers with decimal fractions.

Thousands	Hundreds	Tens	Units		Tenths	Hundredths	Thousandths	Ten-thousandths
			4	.	1	2	5	
5	6	0	9	.	6	8		
			3	.	6	0	4	5
			7	.	2	5		
5	6	2	4	.	6	5	9	5

3. Add: 4.27, 15.004, .9007, and 23.

Find the sum of:

4. .275, .425, .3924, and 40.82.
5. .001, .0002, 831.41, and 2642.893.
6. .0072, .2394, .00801, and 1.3214.
7. 13.279, 3.00046, 742.0003.
8. .00049, 3.24, 15, 42.6, 324.42037.
9. 49.327, .458, 8317.05, 341.875, 32.4962.
10. 700.372, 894.0009, .347, .00082, 5370.006.
11. 560.379, .45687, 350.0036, 7.074, 52.257.

SUBTRACTION OF DECIMALS

Write units of the same order and the decimal points in the same columns. Subtract as in simple subtraction.

1. Find the difference between .47 and .35. Here both decimals are of the same order, and .47 is the larger of the two.

$$\begin{array}{r} .47 \\ \underline{.35} \\ .12 \end{array} \qquad \text{Proof: } .47 - .35 = \tfrac{47}{100} - \tfrac{35}{100} = \tfrac{12}{100} = .12$$

2. Find the difference between .888 and .9.

$$\begin{array}{r} .900 \\ \underline{.888} \\ .012 \end{array} \qquad \begin{array}{l} .9 = .90 = .900. \\ \text{Proof: } .9 = \tfrac{9}{10} = \tfrac{90}{100} = \tfrac{900}{1000}. \quad .888 = \tfrac{888}{1000} \\ \tfrac{900}{1000} - \tfrac{888}{1000} = \tfrac{12}{1000} = .012 \end{array}$$

3. Find the difference between 1 and .875.

$$\begin{array}{r} 1.000 \\ \underline{.875} \\ .125 \end{array} \qquad \begin{array}{l} 1 = 1.000 \text{ since } 1 = \tfrac{10}{10} = \tfrac{100}{100} = \tfrac{1000}{1000} \\ \text{Proof: } \tfrac{1000}{1000} - \tfrac{875}{1000} = \tfrac{125}{1000} = .125 \end{array}$$

Find the difference between:

4. 56.249 and 5.218

5. 9.005 and 7.462

6. 53.316 and 5.086

7. .799 and .6

8. 6.047 and 5.986

9. 850.007 and 270.23

10. 10.872 and 9.981

11. 2. and 1.7689

12. 102.1 and 99.0021

13. 84. and 4.884

14. John Rodman paid one and four thousandths dollars per share for stock in a manufacturing company, and sold it at one and five tenths dollars per share. Did he make or lose money? How much per share?

DECIMALS

Express in decimal form the following fractions:

1. 6 tenths.
2. 6 hundredths.
3. 6 thousandths.
4. 46 hundredths.
5. 62 ten-thousandths.
6. 674 millionths.
7. 605 ten-millionths.
8. 6901 hundredths.
9. 56 thousandths.
10. 706 thousandths.
11. 376 thousandths.
12. 625 ten-thousandths.
13. 316 hundredths.
14. 80060 thousandths.
15. 306 tenths.
16. 5046 ten-thousandths
17. 81 hundred-thousandths.
18. 9 millionths.

Express in decimal form the following fractions:

19. $\frac{9}{10}$.
20. $\frac{24}{100}$.
21. $\frac{4}{100}$.
22. $\frac{42}{100}$.
23. $\frac{125}{1000}$.
24. $\frac{22}{1000}$.
25. $\frac{14}{100}$.
26. $\frac{7}{1000}$.
27. $\frac{102}{1000}$.
28. $\frac{20}{100}$.
29. $\frac{8}{1000}$.
30. $\frac{30}{1000}$.
31. $\frac{422}{10000}$.
32. $\frac{296}{10000000}$.
33. $\frac{89287}{10000000}$.
34. $\frac{900}{1000}$.

Some common fractions may be very easily changed to decimals. We know that a quarter of a dollar of money is twenty-five cents. Cents mean hundredths of a dollar.

$\$\frac{1}{4} = 25¢ = \$.25$ $\quad \$\frac{1}{2} = 50¢ = \$.50$ $\quad \$\frac{3}{4} = 75¢ = \$.75$

$\frac{1}{4} = .25$, since $\frac{1}{4}$ of $100 = 25$, and $\frac{1}{4}$ of $\frac{100}{100} = \frac{25}{100}$.

$\frac{1}{2} = .50$, since $\frac{1}{2}$ of $100 = 50$, and $\frac{1}{2}$ of $\frac{100}{100} = \frac{50}{100}$.

$\frac{1}{2} = .5$, since $\frac{1}{2}$ of $10 = 5$, and $\frac{1}{2}$ of $\frac{10}{10} = \frac{5}{10}$.

$\frac{1}{5} = .2$, since $\frac{1}{5}$ of $10 = 2$, and $\frac{1}{5}$ of $\frac{10}{10} = \frac{2}{10}$.

VALUE OF ONE

$1 = \frac{10}{10}$ or $1.0 = \frac{100}{100}$ or $1.00 = \frac{1000}{1000}$ or $1.000 = \frac{10000}{10000}$ or 1.0000, etc.

DECIMALS AND COMMON FRACTIONS

Every common fraction may be expressed in an equivalent decimal fraction, and every decimal may be expressed as a common fraction.

$\frac{1}{2}$ = 50 per cent of anything, since $\frac{1}{2}$ of 100 is 50; and in percentage we think of everything as having 100 equal parts.

$\frac{1}{4} = 25\%$ $\frac{1}{8} = 12\frac{1}{2}\%$ $\frac{1}{5} = 20\%$ $\frac{1}{10} = 10\%$

50% may be written in decimals: $50 = .5$; $25\% = .25$; $12\frac{1}{2}\% = .125$, since $12\% = .12$, and $\frac{1}{2}$ of $.01 = \frac{1}{2}$ of $.010$, and $\frac{1}{2}$ of $.010 = .005$.

$20\% = .20 = .2$. $10\% = .10 = .1$. Since all these things we know from percentage, we can now say

$\frac{1}{2} = .5$ $\frac{1}{4} = .25$ $\frac{1}{8} = .125$ $\frac{1}{5} = .2$ $\frac{1}{10} = .1$

We studied all these facts when we learned the aliquot parts of 100. We are now expressing just the same facts, but in a different way.

There are a good many decimal fractions which are easy to change to common fractions.

$.1 = \frac{1}{10}$. $.2 = \frac{1}{5}$, because $\frac{2}{10} = \frac{1}{5}$. $.25 = \frac{1}{4}$. $.333+ =$ nearly $\frac{1}{3}$, since $\frac{1}{3}$ of $100 = 33\frac{1}{3}$. $.4 = \frac{2}{5}$, since $\frac{4}{10} = \frac{2}{5}$. $.5 = \frac{1}{2}$. $.666+ = \frac{2}{3}$. $.75 = \frac{3}{4}$.

1. Write the common fractions which equal .2; .333; .25; .50; .75; .1; and .375.

2. Mary had $\frac{3}{4}$ of a dollar, and spent 20 cents for a box of strawberries and 30 cents for half a quart of fine ice cream. Can you find the answer to this question by using decimals? Do so, if you think you can.

SUMS

Add the units' column beginning at the bottom,

$$9 + 2 = 11;\ 11 + 7 = 18;\ 18 + 9 = 27$$

27 +, then notice that $5 + 1 + 4 = 10$, hence

$$27 + 10 = 37;\ 37 + 3 = 40;\ 40 + 9 = 49$$

Set 9 in units' place. Begin the second column, or tens', with the 4 tens of 49.

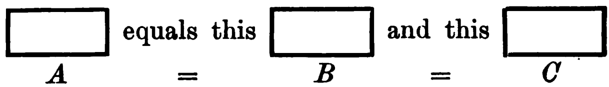
1.	2.	3.	4.	5.	6.	7.	8.
929	363	398	934	838	797	792	394
523	434	629	745	232	698	449	428
394	511	141	526	294	55	998	653
411	146	913	857	167	943	497	298
265	222	834	293	893	55	698	700
399	484	424	632	591	879	262	424
377	878	392	379	954	470	894	787
982	393	550	492	535	346	880	418
879	785	626	115	349	371	334	404

9.	10.	11.	12.	13.	14.	15.	16.
385	385	240	600	968	751	150	956
734	834	452	663	93	936	347	489
269	562	994	765	427	98	244	194
387	983	695	784	749	994	878	934
328	688	363	936	75	986	845	846
493	946	224	878	994	479	732	223
959	51	539	337	96	50	536	938
361	439	446	652	739	654	460	547
349	101	117	343	505	333	875	717

Prove by adding, beginning at the top of each column. Always begin tens' or hundreds' column with the tens or hundreds carried from the preceding column.

EQUATION OF FORMS

Evidently this

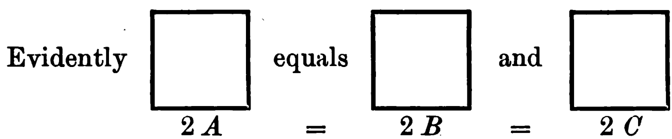


If $A = B$ and $B = C$, then $A = C$.

Things equal to the same things are equal to each other.

1. If 3 apples cost 10¢ and 2 oranges cost the same as 3 apples, how much do the 2 oranges cost?

2. If John and Charles earn together $1 a day, and Thomas earns as much as both John and Charles, how much does Thomas earn?

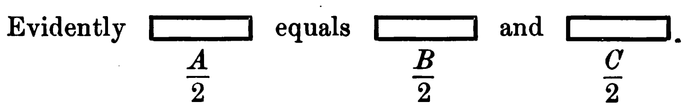


3 A would equal 3 B or 3 C.

Equal multiples of equal things are equal.

3. If Mary and Sally are each 6 yr. old, when Mary is 12 years old, how old will Sally be, if she lives?

4. If one book costs $1, how much will ten $1 books cost?

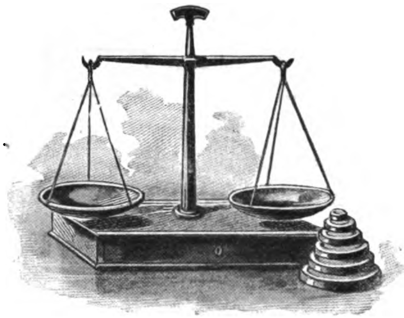


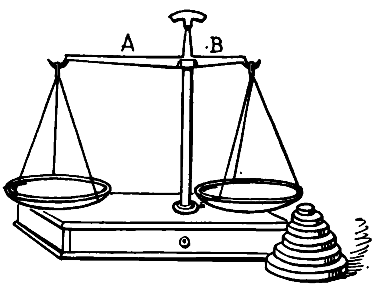
If $\frac{A}{2} = \frac{1}{2}$ of A and $\frac{B}{2} = \frac{1}{2}$ of B, then $\frac{C}{2} = \begin{cases} \frac{1}{2} \text{ of } A \text{ or} \\ \frac{1}{2} \text{ of } B. \end{cases}$

Equal parts of equal things equal each other.

5. If John earns 50¢ a day for ten hours of work, how much will he earn in one fifth of a day?

RATIOS





Scales for Weighing.

The scale arms in the first picture are supported at the center. A weight upon one of the pans is exactly counterbalanced by a weight upon the other.

One pound of the weight balances one pound of sugar.

Four ounces balance four ounces.

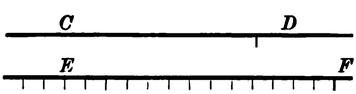
But the scale arms in the second picture are supported at one third of the distance from the right.

$$A = 2B.$$

The ratio of A to B is 2.

A 1-pound weight in A counterbalances 2 pounds of sugar.

The principle of the scale for weighing is **ratio.**



$$C = 3\,D.$$

$$1 \text{ lb. in } C = 3 \text{ lb. in } D.$$

$$E = 16\,F.$$

$$1 \text{ oz. in } E = 1 \text{ lb. in } F$$

The even balance or scale as seen above shows the principle of the equation, both of whose terms are always equal.

EQUATION OF NUMBERS

$6 \times 5 = 3 \times 10$ because $\left.\begin{matrix} 6 \times 5 \\ 3 \times 10 \end{matrix}\right\} = 30$

$12 \times 5 = 6 \times 10$ By what is each quantity multiplied?

$2 \times 5 = 1 \times 10$ By what is each quantity divided?

$2 \times 8 = 4 \times 4$ because $\left.\begin{matrix} 2 \times 8 \\ 4 \times 4 \end{matrix}\right\} = 16$

$6 \times 8 = 12 \times 4$ By what is each quantity multiplied?

$1 \times 8 = 2 \times 4$ By what is each quantity divided?

An equality of quantities is an **equation.**

When we have to find an unknown quantity, sometimes the equation helps us.

$6 \times 5 = x \times 10$ $30 = 10x$ Divide by 10 $3 = x$

$2 \times 8 = x \times 4$ $16 = 4x$ Divide by 4 $4 = x$

$3 \times 4 = x \times 24$ $12 = 24x$ Divide by 24 $\frac{12}{24} = x = \frac{1}{2}$

1. John sold some apples at 2¢ each and received 24¢. How many did he sell?

$x \times 2¢ = 24¢$ Divide by 2¢ $x = 12$

2. Mary found some buttercups on her way to school. She gave 12 to her teacher and $\frac{1}{2}$ of the rest to her school-girl friends. Then she had 3 left. How many butter-cups did she find?

$3 = \frac{x - 12}{2}$, because $3 = \frac{1}{2}$ of all she found less 12.

Multiply both quantities by 2.

$6 = x - 12$ Add 12 to each quantity

$6 + 12 = x - 12 + 12$ $18 = x$

3. John earned some money by selling papers. He gave $\frac{1}{2}$ of it to his mother, bought a cap with $\frac{1}{4}$ of it, and had 50¢ left. How much did he earn?

RATIO AND PROPORTION

$\frac{1}{2}$ of a thing equals $\frac{2}{4}$ of the same thing. $\frac{1}{2} = \frac{2}{4}$.

$\frac{1}{4}$ of a thing equals $\frac{2}{8}$ of the same thing. $\frac{1}{4} = \frac{2}{8}$.

$\frac{1}{2}$ is a ratio. It is the ratio of 1 to 2.

$\frac{2}{4}$ is a ratio. It is the ratio of 2 to 4. ● ● to ⁚ ⁚

These ratios are equal, since $\frac{1}{2} = \frac{2}{4}$.

So we can say, The ratio of 1 to 2 equals the ratio of 2 to 4.

Expressed briefly, we say, 1 is to 2 as 2 is to 4.

We can write this: $1:2 = 2:4$.

This last form, $1:2 = 2:4$, is a **proportion.**

A proportion is an equality of ratios.

1. Explain this form: $1:4 = 2:8$.
2. Prove that $1:3 = 2:6$ and $1:5 = 2:10$.
3. Prove that $2:3 = 4:6$ and $3:4 = 6:8$.

We know that $\frac{2}{3} = \frac{4}{6}$ and that $\frac{3}{4} = \frac{6}{8}$.

4. Draw squares to show these ratios and proportions.

USEFULNESS OF PROPORTION

If we have the ratio of 2 to 4 and the number 6, and wish to know what number is to 6 as 2 is to 4, we can find it by proportion.

● ● is to ⁚ ⁚ as what is to ⁝ ⁝ $2:4 = ?:6$

We find it very helpful in arithmetic to have another sign for ?. This sign is x, the unknown quantity.

$2:4 = x:6$. Find x.

$2 = \frac{1}{2}$ of 4. $x = \frac{1}{2}$ of 6. But $\frac{1}{2}$ of $6 = 3$. $x = 3$.

$2:4 = 3:6$. Proportion helps us find unknown quantities.

PROPORTION

1. If a boy built a toy boat 2 inches wide and 8 inches long, and then planned to build another boat of the same shape but 12 inches long, how wide should he make it?

We can find this by proportion:

$2:8 = x:12 \qquad \frac{2}{8} = \frac{x}{12} \qquad \frac{2}{8} = \frac{1}{4}.$

$1:4 = x:12 \qquad \frac{1}{4} = \frac{x}{12} \qquad \frac{1}{4} \text{ of } 12 = 3.$

$2:8 = 3:12 \qquad \text{The ratio of } \left\{\begin{array}{l} 1 \text{ to } 4 \\ 3 \text{ to } 12 \end{array}\right\} \text{ is } \frac{1}{4}$

The boat should be 12 inches long and 3 inches wide. Draw on the blackboard these two sizes, 2×8 and 3×12, for the boats. Do the forms seem alike in shape? Is the ratio of length to breadth in each form the same?

2. An older boy built a large boat, 30×6 inches. His brother wished to build a smaller boat, 15 inches long, to look like it. How wide did he make it?

We can find this by proportion:

$30:6 = 15:x \qquad \frac{30}{6} = \frac{15}{x} \qquad \frac{30}{6} = \frac{5}{1} \text{ or } 5$

$5:1 = 15:x \qquad \frac{5}{1} = \frac{15}{x} \qquad 15 = 5 \times 3$

$30:6 = 15:3 \qquad \text{The ratio of } \left\{\begin{array}{l} 30 \text{ to } 6 \\ 15 \text{ to } 3 \end{array}\right\} \text{ is } 5$

The boat was 3 inches wide.

3. Find the unknown x in these proportions:

a. $4:3 = x:6$.	*e.* $3:4 = 9:x$.	*i.* $5:4 = x:8$.
b. $2:3 = 4:x$.	*f.* $1:2 = 6:x$.	*j.* $2:1 = 14:x$.
c. $1:5 = 2:x$.	*g.* $3:5 = 12:x$.	*k.* $3:2 = x:10$.
d. $2:5 = x:10$.	*h.* $2:4 = 5:x$.	*l.* $2:3 = x:9$.

PROPORTION

If $2:8=x:12$, what is x?

$$\frac{2}{8}=\frac{x}{12} \quad \frac{2}{8}=\frac{1}{4} \quad \frac{1}{4}=\frac{x}{12} \quad 1=\frac{4x}{12}$$

$$\frac{4x}{12}=\frac{x}{3} \quad 1=\frac{x}{3} \quad 3=x \quad 2:8=3:12$$

The ratio of $2:8$ is $\frac{1}{4}$ and of $3:12$ is $\frac{1}{4}$.

1. How do we know that $\frac{2}{8}=\frac{1}{4}$? Show this by a drawing, if necessary.
2. Why can we say here that $\frac{1}{4}=\frac{x}{12}$? and $1=\frac{4x}{12}$?
3. What do we think in order to make $\frac{4x}{12}=\frac{x}{3}$?
4. What truth makes us know next that $1=\frac{x}{3}$?
5. How did we take the next step, $3=x$?
6. Why do we write in the answer 3 for the x?

We can always do anything to one side or member of an equation when we do the same thing to the other member.

The sign = shows the two equal quantities of the equation.

a. If $3:9=x:15$, what is x?

b. If $2:6=x:3$, what is x?

c. If $2:6=3:x$, what is x?

d. If $3:9=4:x$, what is x?

e. John had 6 marbles and Walter had 3; William had 12 and Samuel had as many in comparison with William as Walter had in comparison with John. How many marbles had Samuel?

f. $10\%:50\%=\$5:\$x$. Find $\$x$.

PERCENTAGE

When we say 10 per cent, or write 10%, we mean 10 : 100. 15% means 15 : 100, or $\frac{15}{100}$. What do 25%, $12\frac{1}{2}$%, 30%, 100%, 75%, mean?

Percentage is a ratio of one thing to some other thing, supposed to be divided into 100 parts.

1. A boy had $3 and gave 15% to his brother. How much did he give him?

We can find the answer by proportion and equation.

dollars per cent		
$x : 3 = 15 : 100$	$\frac{x}{3} = \frac{15}{100}$	Multiply by 3
$x = \frac{45}{100}$	$\frac{45}{100}$ dollars = $.45 = 45¢	

2. Another boy had 200 pigeons. He gave 20% to a friend and sold 30% to a neighbor. How many had he left?

$$20\% + 30\% = 50\%$$

pigeons per cent		
$x : 200 = 50 : 100$	$\frac{50}{100} = \frac{x}{200}$	Multiply by 100
$50 = \frac{x}{2}$	Multiply by 2	$100 = x$

3. Find the unknown quantity:

a. 20% of $x = 50$.
b. 15% of $x = 30$.
c. $12\frac{1}{2}$% of $x = 4$.
d. $66\frac{2}{3}$% of $x = 20$.
e. 25% of $x = 5$.
f. 10% of x = $2.
g. 5% of x = $200.
h. 50% of x = $1200.
i. 75% of x = $750.
j. $33\frac{1}{3}$% of x = $1000.

4. A man borrowed some money at 6% interest for a year. At the end of the year he paid $12 as interest. How much money did he borrow?

ADDITION SUMS

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
24	72	72	83	56	18	37	68	58	76
33	46	58	75	74	42	85	45	76	40
41	19	19	64	10	11	42	84	32	38
16	60	30	38	49	79	17	12	49	9

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
50	96	32	98	90	63	98	93	81	39
99	50	63	63	73	53	32	49	78	62
29	89	96	83	91	93	98	97	34	89
12	96	32	30	63	43	94	93	28	78
57	33	80	95	79	89	78	53	89	97
39	52	52	95	26	26	93	35	30	63
92	30	16	67	39	38	89	44	63	94
91	43	34	34	20	39	98	69	84	31
25	10	23	39	38	41	23	33	92	33
86	22	59	46	80	79	89	47	24	14

21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
40	97	90	23	38	854	78	593	332	99
72	72	90	25	8	928	83	329	763	258
94	93	55	97	39	343	34	432	548	389
33	38	33	96	33	878	90	971	932	493
79	95	97	99	98	767	6	338	800	778
62	93	89	65	98	333	30	432	225	893
34	29	32	89	92	234	83	842	197	939
90	37	86	93	80	843	97	910	341	933
80	87	90	80	79	112	98	589	939	422
80	56	78	75	89	778	20	780	122	224

REVIEW

1. If the circumference of a wheel is 16 feet, how many feet will the wheel travel in 375 revolutions?

2. If a train moves at the rate of 43 miles an hour, how many miles will it travel in 113 hours?

3. How many cubic inches are there in 23 cubic feet? in 40 cubic feet? in 50 cubic feet?

4. A city containing 24,500 inhabitants has provisions for 17 weeks. How many people could live for one week on those provisions?

5. If a man spends $13 a week, how many dollars will he spend in 52 weeks? in 104 weeks?

6. If a man can walk 4 miles an hour, how many miles can he walk in 13 hours?

7. How many cubic feet are there in 391 cubic yards? in 400 cubic yards?

8. If one man can lift 521 pounds' weight, how many pounds can 357 equally strong men lift?

9. If a man walks 37 miles in a day, how many miles does he walk in 313 days? in 365 days?

10. If 39 men can do a certain amount of work in 23 days, how many days will one man take to do the work?

11. If an acre of wheat yields 29 bushels, how many bushels will 149 acres yield?

12. What will 217 horses cost at $106 each?

13. If a city contains 97 streets, and if there are, on an average, 304 houses on each street and 7 persons in each house, what is the population of the city?

14. Since sound travels about 1100 feet per second, how far does it travel in 5 seconds?

REVIEW

1. How many days are there in 48 weeks?

2. Find the sixth part of eighty-four thousand three hundred seventy.

3. Divide eight hundred ninety-seven thousand sixty-three by six.

4. A book has four hundred seven pages. How many pages do 750 such books contain?

5. If a man travels one hundred five miles a day, how far will he travel in two hundred forty-six days?

6. How many times is nine contained in three hundred sixty thousand four hundred seventy-eight?

7. How many sheets are there in two hundred seventeen reams of paper?

8. How many pieces, each nine inches long, can be cut from a roll of wire 3109 inches in length?

9. How many boards, each five feet long, will reach thirty thousand seven hundred thirty feet?

10. How many yards is it from Alpha to Newton, a distance of 462 miles?

11. $120{,}050 \div 7 = ?$

12. If an army consume five thousand seven hundred eighty-nine pounds of bread a day, how many pounds of bread will be required for 287 days?

13. If a township contains 23 classes, and each class averages 37 pupils, how many children are there in all the classes of the township?

14. What was the price of a horse offered for sale at 15% less than $200?

15. There were one hundred twenty-six bales of cotton in a ship, and each bale weighed five hundred pounds. How many pounds of cotton were there in the ship?

VARIATIONS OF PROBLEMS

I. A boy raised 40 hens from the chickens hatched by 4 hens sitting on an average of 13 eggs each. He paid an average of 2¢ each for the eggs; and for food for all he spent $4; he received an average of 50¢ each for the 40 hens. What was his gain?

Substitute

	A for 40 hens	B for 13 eggs	C for 2¢ each	D for 50¢ each
1.	35	14	3¢	60¢
2.	30	15	1¢	25¢
3.	25	12	4¢	75¢
4.	20	11	5¢	$1
5.	50	16	10¢	$3

6. Write the problem, using this combination of facts: 1 A, 2 B, 3 C, 4 D.

7. Write it, using this combination: 3 A, 4 B, 5 C, 2 D.

II. A girl helped her aunt make 12 mince pies. Her aunt sold them for 35¢ each, and gave her niece $\frac{1}{7}$ of the price for each pie as her share. How much did the girl receive?

	E Substitute for 12 pies	F for 35¢	G for $\frac{1}{7}$
1.	15	25¢	$\frac{1}{5}$
2.	10	50¢	$\frac{1}{10}$
3.	18	40¢	$\frac{1}{8}$
4.	24	20¢	$\frac{1}{4}$
5.	36	30¢	$\frac{1}{6}$

VALUE OF MONEY

1. Make a list of things and services money will buy. Let each boy and girl in the class make suggestions. The articles may be classified as Necessities, Comforts, Luxuries; the services as Business, Professional, Domestic.

2. Make lists of the things one dollar will buy; one quarter; one dime; a ten-dollar bill.

3. Develop the ratio of money to articles and services.

$1 buys a nice book, a boy's hat, six pounds of beef.

$2 pays for the doctor's visit, for an 80-mile railroad ride.

$3 buys a pair of men's shoes, or pays the dentist for filling a tooth.

$4 buys a parlor chair, or rents a piano for a month.

$5 buys a girl's dress, or a small boy's suit of clothes.

$20 pays a month's rent for a six-room house "with improvements."

$100 buys a horse, or pays a 2% tax on a $5000 house.

4. Show $1, $2, $5, $10 bills to the pupils; also the $10 gold eagle; and ask their ratios to small coins.

5. Discuss penny-and-nickel-a-day saving and its results; also discuss penny-a-day spending for trifles.

6. Explain savings bank interest: $3, $3½, or $4 a year for each $100.

7. Discuss good books as good investments even of money. Explain even the money-earnings of such investments.

8. $1000 a year is a fair income for a man who has had 15 years' education. $500 a year is a fair income for a man with less than 5 years' education. $500 is 5% interest on what amount of money? Why is a good education usually worth at least as much as $25,000?

Call these subjects "the purchasing power of money."

REVIEW

1. Two men travel in opposite directions; one goes 31 miles a day, and the other 42. How far apart will they be in 12 days?

2. How long can 6 men live on an amount of food that will last one man in the Arctics 762 days? Would an exploration party find it useful to know these facts?

3. Five hundred four dollars are to be divided equally among seven men. How many dollars should be given to each man?

4. A tank contains 16,632 cubic inches of water. How many gallons of water are in the tank?

5. With horses at \$100 each, cows at \$50 each, sheep at \$12.50 each, how many of these different animals will \$1000 buy, if one buys all of the same kind?

6. If $3 : 15 = x : 20$, what is x?

7. Wood was \$4 a cord. A dealer paid for \$1000 worth. How many cords did he buy?

8. In a battle in Cuba, 67 soldiers were killed out of 823 engaged. How many men were left alive?

9. A steamer travels 10,082 miles in one year, and 8241 in a second year. How many more miles did the steamer travel in the first year than in the second?

10. A boy filled ninety-nine boxes with oranges, packing in each box three hundred ninety-nine oranges. How many oranges did he pack in all the boxes?

11. A room was 12 ft. high, 24 ft. wide, and 32 ft. long. How many cubic feet did it contain?

12. If each boy breathed 30 cu. ft. of air a minute, how long would the air be fresh with one boy in the room? with 50 boys?

13. If $2 : 9 = 4 : x$, what is x?

REVIEW

Add together:

1. 4.13295, 42.171, .3721, .0064, .6, 25.
2. 3010.3, 21.747, .006887, .96, 325.37, 141.2.
3. .0065, 65, .065, 6.5, .000065, .65.
4. 16.5, 176.1105, 11.0012, 18.75, 11.5, 14.123.
5. .813, 72.5, 32.071, 2.1574, 371.4, 2.75.
6. 7.1, 3275, 3.2675, 1.00005, 14.7, .001.

Find the prime factors:

7.	8.	9.	10.	11.
84	90	100	112	117
1760	1690	2520	2432	121
75	98	147	495	132
135	168	196	125	144
385	624	945	231	1575
1296	1331	1617	1188	1050

12. Multiply each number in column A by each number in B.

A Multiplicands	B Multipliers
a. 95,080	*f.* 870
b. 40,009	*g.* 706
c. 76,900	*h.* 918
d. 81,118	*i.* 118
e. 48,484	*j.* 896

Prove each answer by treating each product as a dividend and each multiplier as a divisor.

REVIEW

1. If a man works nine hours a day, how many hours does he work in 301 days?

2. A boy traveled in 7 days in a train 2870 miles. How far did he travel each day?

3. If 8 men can do a piece of work in 10 days, in how many days and parts of days can 24 men do it working at the same rate?

4. A farmer stored in his barns 100 tons of hay for the winter. He kept 40 cows and horses. At the end of the winter 40 tons were left. How many tons each did the cows and horses eat on the average?

5. A man had $500 to lend. One person offered him 4% on it, and another offered 5%. How much money, more or less, would he make in one year by lending at the 5% rate?

6. What is a proportion? an equation?

7. Add .02, .25, .13.

8. Subtract 1.29 from 2.453.

9. What decimal equals the common fraction $\frac{1}{8}$?

10. Read 1001.2324 : 12 : : 00456 : 1000 : : 0275.

11. Give the tables for **counting, time, avoirdupois weight,** and **dry** and **liquid measures.**

12. What is the circumference of a circle 10 ft. in diameter?

13. What is the area of a right-angled triangle 5 ft. by 24 ft.?

14. How much is 5×6 divided by 2×3?

TABLES

Dry Measure

2 pints	= 1 quart	2 pt.	= 1 qt.
8 quarts	= 1 peck	8 qt.	= 1 pk.
4 pecks	= 1 bushel	4 pk.	= 1 bu.

Liquid Measure

4 gills	= 1 pint	4 gi.	= 1 pt.
2 pints	= 1 quart	2 pt.	= 1 qt.
4 quarts	= 1 gallon	4 qt.	= 1 gal.

Time Measure

60 seconds	= 1 minute	60 sec.	= 1 min.
60 minutes	= 1 hour	60 min.	= 1 hr.
24 hours	= 1 day	24 hr.	= 1 da.
7 days	= 1 week	7 da.	= 1 wk.
12 months	= 1 year	12 mo.	= 1 yr.
30 days count as	1 month usually	30 da.	= 1 mo.
365 days count as	1 year usually	365 da.	= 1 yr.

Length Measure

12 inches	= 1 foot	12 in.	= 1 ft.
3 feet	= 1 yard	3 ft.	= 1 yd.
$5\frac{1}{2}$ yards	= 1 rod	$5\frac{1}{2}$ yd.	= 1 rd.
5280 feet	= 1 mile	5280 ft.	= 1 mi.
1760 yards	= 1 mile	1760 yd.	= 1 mi.

Weight Measure

16 ounces	= 1 pound	16 oz.	= 1 lb.
100 pounds	= 1 hundredweight	100 lb.	= 1 cwt.
2000 pounds	= 1 ton	2000 lb.	= 1 T.

2240 pounds = 1 long ton (used for weighing coal at the mines and for merchandise at United States Custom Houses).

U. S. Money

5 cents	= 1 nickel	5¢	= 1 n.
10 cents	= 1 dime	10¢	= 1 d.
100 cents	= 1 dollar	100¢	= $1

TABLES

Counting Measure

12 units	= 1 dozen	12	= 1 doz.
12 dozen	= 1 gross	12 doz.	= 1 gr.
12 gross	= 1 great gross	12 gr.	= 1 G. gr.
20 units	= 1 score	20	= 1 sc.

Square Measure

144 square inches	= 1 square foot	144 sq. in.	= 1 sq. ft.
9 square feet	= 1 square yard	9 sq. ft.	= 1 sq. yd.
$30\frac{1}{4}$ square yards	= 1 square rod	$30\frac{1}{4}$ sq. yd.	= 1 sq. rd.
160 square rods	= 1 acre	160 sq. rd.	= 1 A.
43,560 square feet	= 1 acre	43,560 sq. ft.	= 1 A.
640 acres	= 1 square mile or section		

Cubic Measure

1728 cubic inches	= 1 cubic foot	1728 cu. in.	= 1 cu. ft.
27 cubic feet	= 1 cubic yard	27 cu. ft.	= 1 cu. yd.

Cord Measure

16 cubic feet	= 1 cord foot	16 cu. ft.	= 1 cd. ft.
128 cubic feet	= 1 cord	128 cu. ft.	= 1 cd.

Fractions and Per Cents

$\frac{1}{2} = 50\%$.	$\frac{1}{3} = 33\frac{1}{3}\%$.	$\frac{1}{4} = 25\%$.
$\frac{1}{5} = 20\%$.	$\frac{1}{6} = 16\frac{2}{3}\%$.	$\frac{1}{8} = 12\frac{1}{2}\%$.
$\frac{1}{10} = 10\%$.	$\frac{1}{12} = 8\frac{1}{3}\%$.	$\frac{1}{16} = 6\frac{1}{4}\%$.

ORAL TEST OF SUCCESS

1. What is the ratio of 1 bu. to 2 qt.? of 4 oz. to 1 lb.?

2. A boy who takes 16 breaths a minute would take how many breaths in an hour?

3. What is the area of a right-angled triangle whose base is 3 in. and height is 6 in.?

4. Read in words these numbers: 105 432 862 100; 835.6245; MCMIX.

5. What is $16\frac{2}{3}\%$ of \$900? of \$3600?

6. Give two fractions equal to $\frac{2}{5}$.

WRITTEN TEST OF SUCCESS

7. If $3 : 8 = x : 24$, what is x?

8. *a.* Express $10\frac{3}{4}$ in decimals.
   b. Express 5.05 as a mixed number.

9. Divide $8 \times 9 \times 5 \times 3$ by 72, using cancellation.

10. A load of hay weighed 4300 lb., the horses 3100 lb., the wagon 1800 lb., and the driver 175 lb. Find the total weight expressed in T., cwt., and lb.

11. A stationery dealer in one year sold 16 gross of pens at 1¢ each. What was the amount of the sales?

12. A man with \$100 in his purse paid out the following amounts: $\$2\frac{1}{2}$, 30¢, a dime, a quarter, \$12.75, three nickels, five half-dollars, and a X-dollar bill. How much money was left in the purse?

Boys and girls who are able to answer correctly all of twelve such questions as these and to add correctly such columns of figures as those on pages **18**, **36**, **126**, and **134** know this book sufficiently well to begin another.

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